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FACULTY OF ECONOMICS

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- Introduction to QS
- In this course we will cover
  1) Linear Algebra
  2) Mathematical Analysis
  3) Optimization, Probability, Statistics
- Today: Sets, Real Numbers, Functions, Matrices, Systems of Linear Equations

- Set is a collection of objects

$A = \{x \mid P(x)\} \Leftrightarrow A$ is a set of objects x so that property $P(x)$ is true

$x \in A \Leftrightarrow x$ belongs to set A

$x \notin A \Leftrightarrow x$ doesn't belong to set A

- Examples

$\mathbb{N} = \{1, 2, 3, \dots\}$ Natural numbers

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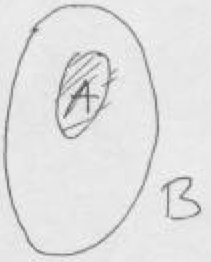
$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ Integer Numbers

$\mathbb{Q} = \{\frac{p}{q} \mid p \in \mathbb{Z},\ q \in \mathbb{N}\}$ Rational Numbers

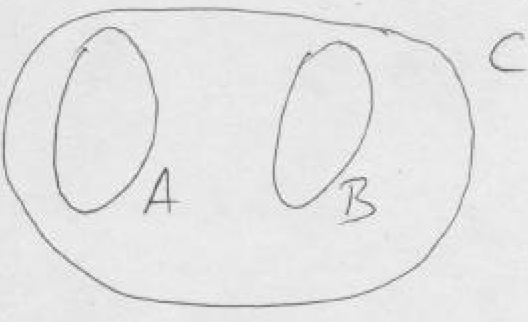
– Subset: $A \subseteq B \iff (\forall x \in A)\ x \in B$

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q}$

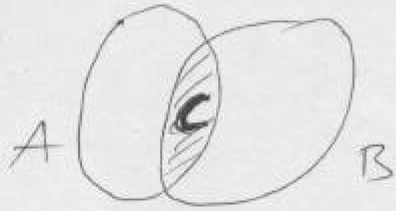


A is a subset of B

– Union of sets: $C = A \cup B \iff (\forall c \in C)(c \in A \vee c \in B)$



– Intersection of sets: $C = A \cap B \iff (\forall c \in C)(c \in A \wedge c \in B)$



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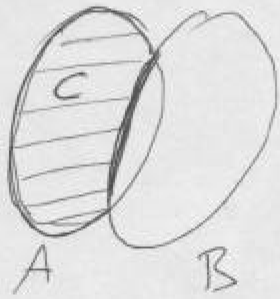
- Difference of sets: $C = A \setminus B \iff$
  $(\forall c \in C)(c \in A \wedge c \notin B)$



- Set of Real Numbers $\mathbb{R}$

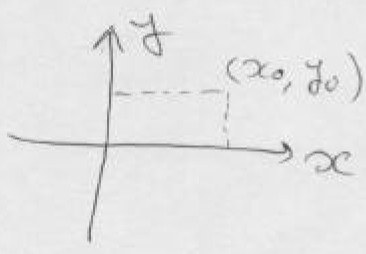
  $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} (\subseteq \mathbb{C})$

  Real Numbers form a field
  $(\mathbb{R}, +, \cdot)$

- Product of sets: $C = A \times B$

  $C = \{(a,b) \mid a \in A, b \in B\}$

  set of ordered pairs (a,b)



$\mathbb{R}^2 = \{(x,y) \mid x, y \in \mathbb{R}\}$

coordinate system

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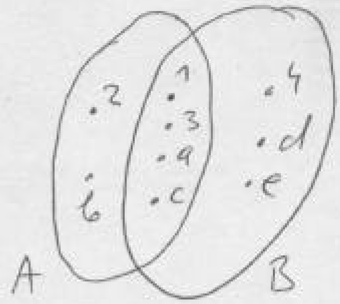
– Examples:

1) $A = \{1, 2, 3, a, b, c\}$, $B = \{1, 3, 4, a, c, d, e\}$

$A \cup B = \{1, 2, 3, 4, a, b, c, d, e\}$

$A \cap B = \{1, 3, a, c\}$

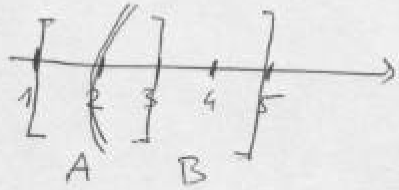
$A \setminus B = \{2, b\}$ $\quad B \setminus A = \{4, d, e\}$



2) $A = [1, 3]$ $\quad B = (2, 5]$

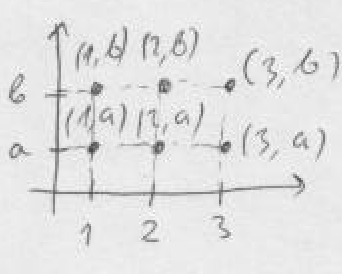
$A \cup B = [1, 5]$ $\quad A \cap B = (2, 3]$

$A \setminus B = [1, 2]$ $\quad B \setminus A = (3, 5]$



3) $A = \{1, 2, 3\}$ $\quad B = \{a, b\}$

$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$



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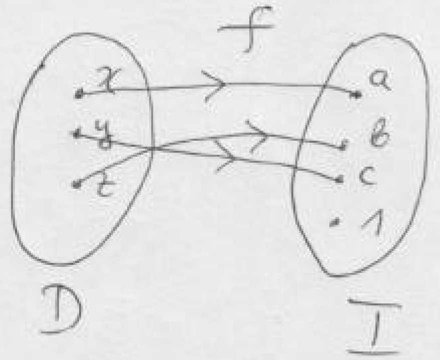
- Function f is a map from a domain set D to an image set I:

$$f : D \to I$$

$$(\forall d \in D)(\exists_1 f(d) \in I)$$



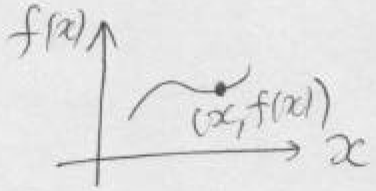
- Real functions of a real variable

$$f : \mathbb{R} \to \mathbb{R}$$

$$(\forall x \in \mathbb{R})(\exists_1 f(x) \in \mathbb{R})$$

Examples: $f(x) = x$, $f(x) = x^2$, ...

- Graph of the function



properties: minima, maxima, stationary points, trends

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- Elementary functions:

1) Linear function

$$f(x) = ax + b$$

2) Polynomial function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \equiv P_n(x)$$

3) Rational function

$$f(x) = \frac{P_n(x)}{Q_m(x)} = \frac{a_n x^n + \cdots + a_0}{b_m x^m + \cdots + b_0}$$

4) Exponential function

$$f(x) = e^x$$

5) Logarithm

$$f(x) = \log_a x$$

6) Trigonometric functions

$$f(x) = \sin x \qquad f(x) = \cos x$$

$$f(x) = \tan x \qquad f(x) = \cotan x$$

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7) Inverse Trigonometric functions

$f(x) = \text{Arsin}\, x$ $f(x) = \text{Arcos}\, x$

$f(x) = \text{Artan}\, x$ $f(x) = \text{Arcotan}\, x$

8) Hyperbolic functions

$f(x) = \sinh x$ $f(x) = \cosh x$

$f(x) = \tanh x$ $f(x) = \text{cotanh}\, x$

9) Inverse Hyperbolic functions

$f(x) = \text{Arsinh}\, x$ $f(x) = \text{Arcosh}\, x$

$f(x) = \text{Artanh}\, x$ $f(x) = \text{Arcotanh}\, x$

– We will now review basic properties of Microsoft Excel Spreadsheet Program, including entering formulas and representing data using charts

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- Matrix is a table of objects; the size of a matrix is $m \times n$, m is the number of rows, n is the number of columns

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

- We will work with matrices $\in \mathbb{R}^{mn}$, i.e. matrices with $a_{ij} \in \mathbb{R}$

- Addition and subtraction of matrices of the same dimensions

$$A \pm B = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \pm \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ b_{m1} & \cdots & b_{mn} \end{bmatrix} = \begin{bmatrix} a_{11} \pm b_{11} & \cdots & a_{1n} \pm b_{1n} \\ \vdots & & \vdots \\ a_{m1} \pm b_{m1} & \cdots & a_{mn} \pm b_{mn} \end{bmatrix}$$

$$+ : \mathbb{R}^{mn} \times \mathbb{R}^{mn} \to \mathbb{R}^{mn}$$

$$- : \mathbb{R}^{mn} \times \mathbb{R}^{mn} \to \mathbb{R}^{mn}$$

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– Multiplication

$\cdot : \mathbb{R}^{mn} \times \mathbb{R}^{np} \to \mathbb{R}^{mp}$

$A \cdot B = C$

condition: #columns(A) = #rows(B)

#rows(A) = #rows(C)

#columns(B) = #columns(C)

$$A \cdot B = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & \dots & b_{1p} \\ \vdots & & \\ b_{n1} & \dots & b_{np} \end{bmatrix} =$$

$$= C = \begin{bmatrix} a_{11}b_{11} + \dots a_{1n}b_{n1} & \dots\dots & a_{11}b_{1p} + \dots a_{1n}b_{np} \\ \vdots & & \\ a_{m1}b_{11} + \dots a_{mn}b_{n1} & \dots\dots & a_{m1}b_{1p} + \dots + a_{mn}b_{np} \end{bmatrix}$$

i.e. $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$

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- Examples:

1) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 4 & 12 \end{bmatrix}$

2) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$

3) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 1\cdot 0+2\cdot 1 & 1\cdot 2+2\cdot 8 \\ 3\cdot 0+4\cdot 1 & 3\cdot 2+4\cdot 8 \end{bmatrix} = \begin{bmatrix} 2 & 18 \\ 4 & 38 \end{bmatrix}$

- Quadratic Matrices $A \in \mathbb{R}^{nn}$
  only for this type of matrices we can define power of matrix A^k

- Application of matrices:
  Systems of Linear Equations

$$\begin{aligned} x+y+z &= 1 \\ 2x+y+z &= 2 \\ x+2y+z &= 3 \end{aligned} \iff \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{v} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{a}$$

$$Av = a, \quad A \in \mathbb{R}^{33}, \quad v \in \mathbb{R}^{31} \equiv \mathbb{R}^3, \quad a \in \mathbb{R}^3$$

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- Obviously, the solution will be given by
$$v = A^{-1} a$$
if we can define inverse matrix A^{-1} of matrix A → we will do it later.
- We can enter, add, multiply and subtract matrices in Excel

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