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- To resolve an investment issue with quantitative methods, the issue must first be formulated as a specific problem. The best formulation is a version of optimization. It is entirely consistent with general investment objectives to try to devise the ideal portfolio, to select the best combination of projets, to manage an investment to attain the most favourable outcome, etc.

- The capital allocation problem consists of allocating a (usually fixed) budget among a number of investments or projects. We distinguish between capital budgeting treated here and portfolio problems treated later.

- Capital budgeting problems arise when several different projects compete for funding. The projects may differ considerably in their scale, costs, benefits, but, the critical point is that they cannot all be funded because of a budget limitation.

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- Independent Projects

- The projects in this case are independent in the sense that it is reasonable to select any combination of them. Also, the value of any project doesn't depend on another project also being funded.

- Suppose that there are m potential projects. Let b_i be the total benefit (usually the NPV) of the i-th project, and let c_i denote its initial cost. Finaly, let C be the total capital available - the budget. For each $i=1,2,\dots,m$ we introduce the zero-one variable x_i, which is zero if the project is rejected. and one if it is accepted. The problem is then that of solving:

$$\text{maximize} \left(\sum_{i=1}^{m} b_i x_i \right),$$

subject to constraint $\sum_{i=1}^{m} c_i x_i \leq C$,
where $x_i = 0$ or 1, for $i=1,2,\dots,m$.

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- This is so-called zero-one programming problem.
- We will use Solver in MS Excel to solve this type of problems.

- Example: During its annual budget planning meeting, a small computer company has identified 7 proposals for independent projects that could be initiated in the forthcoming year. The projects all require an initial capital outlay in the next year. The company management have available \$500,000 for these projects. The financial aspects of the projects are shown in table. What is the optimal choice of projects?

Project	Initial Outlay (\$1,000)	PV (\$1,000)
1	100	300
2	20	50
3	150	350
4	50	110
5	50	100
6	150	250
7	150	200

The solution:

$x_1 = 1$

$x_2 = 0$

$x_3 = 1$

$x_4 = 1$

$x_5 = 1$

$x_6 = 1$

$x_7 = 0$

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— Interdependent Projects

- Sometimes various projects are interdependent, the feasibility of one being dependant of whether others are undertaken. We formulate a problem of this type assuming that there are several independent goals, but each goal has more than one possible implementation. It is these implementation alternatives that define the projects.

- Example: Suppose a transportation authority wishes to construct a road between two cities. Corresponding different implementations of this goal might detail whether the road were concrete of asphalt, two lanes or four, etc. Another, independent, goal might be the improvement of a bridge, with several different implementations. We will discuss this example later.

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- In general, assume that there are m goals and that associated with the i-th goal there are n_i possible implementations of this goal (n_i projects). Only one project can be selected for any goal. As before, there is a fixed available budget.
- We formulate this problem by introducing the zero-one variables x_{ij} for $i=1,2,\dots,m$ and $j=1,\dots,n_i$. The variable x_{ij} equals one if goal i is chosen and implemented by project j; otherwise, $x_{ij}=0$. The problem is then

$$\text{maximize}\left(\sum_{i=1}^{m}\sum_{j=1}^{n_i} b_{ij}x_{ij}\right),$$

subject to constraints $\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n_i} c_{ij}x_{ij} \le C$,

$$\sum_{j=1}^{n_i} x_{ij} \le 1, \quad \text{for } i=1,2,\dots,m$$

Where $x_{ij}=0$ or 1, for all i and j.

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– Homework XI

1) Suppose that the goals and specific projects shown in table are being considered by the County Transportation Authority. There are three independent goals and a total of 10 projects. The total available budget is \$5 milion. At ~~la~~ most one project can be selected for each goal. Find the optimal selection of projects using Solver in Excel.

	Cost (\$1 milion)	NPV (\$1 milion)
Goal 1: Road betwen Augen and Burger		
Project 1: Concrete, 2 lanes	2	4
Project 2: concrete, 4 lanes	3	5
Project 3: Asphalt, 2 lanes	1.5	3
Project 4: Asphalt, 4 lanes	2.2	4.3
Goal 2: Bridge at Coy Road		
Project 5: Repair existing	0.5	1
Project 6: Add lane	1.5	1.5
Project 7: New structure	2.5	2.5

cont.

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	Cost ($1milion)	NPV ($1milion)
Goal 3: Traffic Control in Downsberg		
Project 8: Traffic Lights	0.1	0.3
Project 9: Turn lanes	0.6	1
Project 10: Underpass	1	2

- Optimal portfolios: Construction of a portfolio of financial securities or any financial assets. We will here discuss optimization of portfolios of fixed-income instruments (those that return at known points in time), more precisely, Cash Matching Problem.
- Suppose that we face a known sequence of future monetary obligations. We wish to invest now so that these obligations can be met as they occur; and accordingly, we plan to purchase bonds of various maturities and use the coupon payments and redemption values to meet the obligations.

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- To formulate the problem, we will first establish a basic time period length, with cash flows occuring at the end of each of these periods. Our obligation is then a stream $y = (y_1, y_2, \ldots, y_n)$, starting one period from now.

- If there are m different kinds of bonds, we will denote the stream associated with one unit of bond type j by $c_j = (c_{1j}, \ldots, c_{nj})$. The present price of bond type j is denoted by p_j. We will denote by x_j the amount of bond type j to be held in the portfolio. The cash matching problem is to find x_j's of minimum total cost that guarantee that the obligations can be met:

$$\text{minimize}\left(\sum_{j=1}^{m} p_j x_j\right),$$

$$\text{subject to } \sum_{j=1}^{m} c_{ij} x_j \geq y_i, \text{ for } i = 1, 2, \ldots, n,$$

$$\text{where } x_j \geq 0, \text{ for } j = 1, \ldots, m.$$

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– Homework XII

1) We wish to match cash obligations over a 6-year period. We select 10 types of bonds, with maturities ranging from 1 year to 6 years. The cash flow structure of each type of bonds, together with the present price of bonds is shown in table. The cash flow stream representing the obligations is $y = (100, 200, 800, 100, 800, 1200)$. Find the optimal portfolio of bonds that will generate cash flows so that the obligations can be met. Use Solver in Excel for optimization.

Year	Bond 1	Bond 2	Bond 3	Bond 4	Bond 5	Bond 6	Bond 7	Bond 8	Bond 9	Bond 10
1	10	7	8	6	7	5	10	8	7	100
2	10	7	8	6	7	5	10	8	107	
3	10	7	8	6	7	5	110	108		
4	10	7	8	6	7	105				
5	10	7	8	106	107					
6	110	107	108							
Price of Bond	109	94.8	99.5	93.1	97.2	92.9	110	104	102	95.2

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- Solutions of HW $\underline{IX}$, assigned on January 5, 2004.

1) If the rate of return is $r = 7\%$, then after $n = 10$ years the original amount of money A will be equal (assuming yearly compounding) $V = (1+r)^n A$. Since $(1+r)^n =$ $= (1+0.07)^{10} \approx 1.96715$, we can see that after 10 years, $V \approx 1.96715 A$, i.e. $V \approx 2A$ with 3.3% error.

If the rate of return is $r = 10\%$, after $n = 7$ years is $V = (1+r)^n A = (1+0.1)^7 A$. Since $(1+r)^n = (1+0.1)^7 \approx 1.94872$, we have $V \approx 2A$, with 5.2% error.

2) For the stream $(-1, 1, 1, 1, 2)$ we have (at $r = 10\%$)

$$FV = -1 \cdot 1.1^4 + 1 \cdot 1.1^3 + 1 \cdot 1.1^2 + 1 \cdot 1.1^1 + 2 \approx 4.18$$

$$PV = -1 + \frac{1}{1.1} + \frac{1}{1.1^2} + \frac{1}{1.1^3} + \frac{2}{1.1^4} \approx 2.85$$

$\frac{FV}{1.1^4} = \frac{4.18}{1.1^4} \approx 2.85 = PV$, so $PV = \frac{FV}{(1+r)^n}$ is valid.

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For the stream $(-2, 1, 1, 1, 3)$ we have (at $r=10\%$)

$$FV = -2 \cdot 1.1^4 + 1 \cdot 1.1^3 + 1 \cdot 1.1^2 + 1 \cdot 1.1^1 + 3 \approx 3.71$$

$$PV = -2 + \frac{1}{1.1} + \frac{1}{1.1^2} + \frac{1}{1.1^3} + \frac{3}{1.1^4} \approx 2.54$$

$$\frac{FV}{1.1^4} = \frac{3.71}{1.1^4} \approx 2.54 = PV, \text{ so } PV = \frac{FV}{(1+r)^n} \text{ is valid.}$$

3) The stream $(-1, 1, 1, 2, 1)$ has (at $r=10\%$)

$$PV = -1 + \frac{1}{1.1} + \frac{1}{1.1^2} + \frac{2}{1.1^3} + \frac{1}{1.1^4} \approx 2.92,$$

while the stream $(-1, 0, 1.1, 1, 2.1)$ (at $r=10\%$) has

$$PV = -1 + \frac{0}{1.1} + \frac{1.1}{1.1^2} + \frac{1}{1.1^3} + \frac{2.1}{1.1^4} \approx 2.09.$$

According to the theorem given at the lecture, these two streams are not equivalent, since their PVs are different.