Mathematics and Modeling for Finance

HW2: Due on October 25, 2006

1) Find the first and the second derivative of the following functions:

a) $f(x) = \ln(x + e^{-x^2})$ b) $f(x) = \sqrt{x^3 + \ln(\frac{1}{x} + 2)}$

2) Find the Hessian matrix for the following f's:

a) $f(x,y) = x^2 + y^2$ b) $f(x,y) = \ln(x+y) + e^{x}$

c) $f(x,y,z) = xyz + \ln(xyz)$ d) $f(x,y,z) = e^{x/y} \ln(y^2 + z^2)$

3) Download historical stock prices for at least 3 companies for the last 10 years and find the corresponding rates of return for each stock using "buy today at the lowest price, sell tomorrow at the highest price" approach. Find the covariance matrix for these 3 assets and then find its Cholesky decomposition. Using this decomposition, simulate 1000 days of behaviour of studied rates of return.

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4) Perform the following matrix operations, both manually and in Excel with

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

a) $(A+C^T)B$ b) BCA c) CAB

5) Find the eigenvalues and normalized eigenvetors for the covariance matrix found in problem #3.

6) Find the variance matrix, its Cholesky decomposition and the corresponding eigenvalues and eigenvetors for the portfolio of stocks from the problem #3 taking into account now just the data for the highest stock prices, i.e. each asset will be represented by its mean value, and the covariance matrix should be calculated for the highest prices values.

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7) Find minima and maxima of the following f's:

a) $f(x) = x^2 + x^3$ b) $f(x) = e^x \sin x$

[Use: $\frac{d}{dx}\sin x = \cos x$, $\frac{d}{dx}\cos x = -\sin x$]

8) Find minima and maxima of the following f's:

a) $f(x, y) = x^4 y^2 + x^2 y^4 - x^2 y^2$

b) $f(x, y, z) = 2x^2 + 2xy + 2xz + 2y^2 + 2yz + 2z^2$

9) Solve the constrained optimization problems for the functions given below, where $g(x, y)$ is constrained to be zero.

a) $f(x, y) = x^2 - y^2$, $g(x, y) = 1 - x - y$

b) $f(x, y) = \frac{x^3}{3} - \frac{3y^2}{2} + 2x$, $g(x, y) = x - y$

10) Solve the Markowitz problem for a set of three uncorrelated assets with $\sigma_1^2 = 1$, $\sigma_2^2 = 1$, $\sigma_3^2 = 1$, and $\overline{r}_1 = 1$, $\overline{r}_2 = 2$, $\overline{r}_3 = 3$, with the portfolio rate of return being fixed at $\overline{r} = 1.8$.

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