NPV Calculations, Annuities, Perpetuities

- Investment is defined as the current commitment of resources in order to achieve later benefits
- If resources and benefits take the form of money, investment is the present commitment of money for the purpose of receiving (hopefully more) money later
- The money to be obtained later is, in some cases, known exactly; however, in most cases, this amount is uncertain
- A broader viewpoint, based on the idea of flows of expenditures and receipts spanning a period of time
- From this viewpoint, the objective of investment is to tailor the pattern of these flows over time to be as desirable as possible

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- If these flows are denominated in cash, we have cash flows, and the series of flows over several periods is termed a cash flow stream

- Example: By taking a loan, it may be possible to exchange a large negative cash flow next month for a cash flow stream containing smaller negative cash flows, and this alternative cash flow stream may be preferred to the original one:

debt : (−1000 €)

loan : (−300 €, −300 €, −300 €, −300 €)

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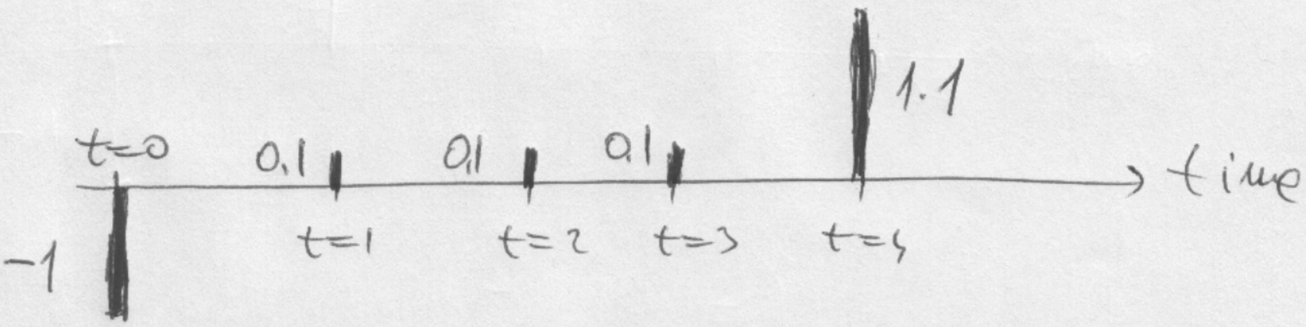
- Usually, the cash flows (positive or negative) occur at known specific dates; the stream can then be described by listing the flow at each of these dates

- Example: Investment over 4 years can be:

$$\begin{array}{ccccc} (-1, & 0.1, & 0.1, & 0.1, & 1.1) \\ t=0 & t=1 & t=2 & t=3 & t=4 \end{array}$$

- Time line representation:



- Typical questions of investment:
  1) Which of two cash flow streams is most preferable?
  2) How much I would be willing to pay to own a given stream?

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3) If I can purchase a share of a stream, how much should I purchase?

4) Given a collection of available cash flow streams, what is the most favourable combination of them (portfolio)?

- Necessary instruments to be able to answer all these questions lie in mathematical analysis of appropriate problems

- Ingredients:

  1) Interest rate

  2) NPV (PV), FV

  3) IRR

- What is interest rate? If you give \$100 in a bank today, in 1 year you will get, say, \$105; therefore, you gained \$5

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- Interest rate is $\frac{\$5}{\$100} = 0.05 = 5\%$

- Economics provides the following insight:
interest rate r = Real risk-free interest rate + Inflation premium + Default risk premium + Liquidity premium + + Maturity premium

- Future value (FV) of a single cash flow – Simple interest rule: principal (A) + interest (rA)

$$FV = A + rA = A(1+r)$$

- If the same principal is invested for N time periods, is its FV_N equals

$$A(1+Nr) \; ?$$

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- Actually, no! This simple rule can be applied if r is very small, and N is small number; or if interest is ~~[illegible]~~ withdrawn after each time period; otherwise, the correct answer is

$$FV_N = A(1+r)^N = A\underbrace{(1+r)\cdot(1+r)\cdots(1+r)}_{N}$$

- Examples: CFA 1-1, CFA 1-2, CFA 1-3

- Compounding can be more frequent; in that case, stated interest rate for a longer time period is denoted r_s; Example: usually, interest rates are described as annual rates, but compounding is done quarterly, or monthly; in that case

$$FV_N = PV\left(1+\frac{r_s}{m}\right)^{mN}$$

wher m is the frequency of compounding over

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the stated period of time

- Examples: CFA 1-4, CFA 1-5

- Continuous compounding is obtained when $m \to \infty$; then ~~discrete~~ compounding can be thought of as to be done in real time

$$FV_N = PV \cdot e^{r_s N} =$$

$$= \lim_{m \to \infty} \left(1 + \frac{r_s}{m}\right)^{mN} \cdot PV$$

- Example: CFA 1-6

- Example: Comparison of compounding frequency of FV, CFA Table 1-1

- This was how to calculate FV for a single cash flows

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- What about cash flow streams?
- Definitions:

1) An annuity is a finite set of level sequential cash flows
2) An ordinary annuity has a first cash flow that occurs one period from now, (0, ...)
3) An annuity due has a first cash flow that occurs immediately (C, ...), $C \neq 0$
4) A perpetuity is a perpetual annuity; never-ending sequential set of cash flows occurring one period from now

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- Example: equal cash flows, ordinary annuity

$$(0, A, A, A, \dots, A)$$
$$t=0 \quad t=1 \quad \dots \quad t=N$$

$$FV_N = A(1+r)^{N-1} + A(1+r)^{N-2} + \dots + A(1+r) + A =$$

$$= A\left[(1+r)^{N-1} + \dots + (1+r)^1 + (1+r)^0\right] =$$

$$= A\,\frac{(1+r)^N - 1}{r}$$

- Example: CFA 1-7

- For unequal cash flows, the calculation must be done separately for each cash flow, and FVs of all of them summed up at the final time

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- PV of a single cash flow

$$FV_N = PV(1+r)^N \Rightarrow PV = \frac{FV_N}{(1+r)^N}$$

- Example: CFA 1-8, CFA 1-9

- If the compounding is more frequent than the stated time period, then

$$FV_N = PV\left(1+\frac{r_s}{m}\right)^{mN} \Rightarrow PV = \frac{FV_N}{\left(1+\frac{r_s}{m}\right)^{mN}}$$

- Example: CFA 1-10

- PV of a series of equal cash flows:

$$PV = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \dots + \frac{A}{(1+r)^N} =$$

$$= A\,\frac{1-\frac{1}{(1+r)^N}}{r}$$

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- Example: CFA 1-11

- PV of a perpetuity

$$PV = A \sum_{t=1}^{\infty} \frac{1}{(1+r)^t} = \frac{A}{r}$$

- Example: CFA 1-14

- Question: Why not calculate FV of a perpetuity?

- solving for interest rates

$$FV_N = PV(1+r)^N \Rightarrow (1+r)^N = \frac{FV_N}{PV} \Rightarrow$$

$$1+r = \sqrt[N]{FV_N/PV} \Rightarrow r = (FV_N/PV)^{1/N} - 1$$

- Example: CFA 1-17, CFA 1-18

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- Solving for the number of periods

$$FV_N = PV(1+r)^N \Rightarrow (1+r)^N = FV_N / PV \Rightarrow$$

$$N \log(1+r) = \log(FV_N / PV) \Rightarrow$$

$$N = \frac{\log(FV_N / PV)}{\log(1+r)}$$

- Example: CFA 1-19

- Solving for the size of annuity payments: CFA 1-20

- Important: the cash flow additivity principle!

	t=0	t=1	t=2
A	0	\$100	\$100
B	0	\$200	\$100
A+B	0	\$300	\$200

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- Comparison of cash flow streams can now be done by comparing their values at the same moment of time, e.g. by comparing their NPVs, or their FVs; the only important point is to calculate their values at the same moment of time

- Example: Which of the two cash streams is most desirable if $r = 10\%$?

  $(-1, 2)$; $(-1, 0, 3)$

- Another approach: IRR (internal rate of return)

- IRR is a rate at which PV of a cash flow stream is equal zero

  $(x_0, x_1, \dots, x_N)$

$$PV = \sum_{k=0}^{N} \frac{x_k}{(1+r)^k}, \quad PV = 0 \Rightarrow r = IRR$$

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- Usually, we introduce $c = \frac{1}{1+r}$, then the equation becomes

$$0 = x_0 + x_1 c + x_2 c^2 + \dots + x_N c^N ,$$

and IRR is $r = \frac{1}{c} - 1$

- Example: Find IRR for $(-2, 1, 1)$.
- Evaluation criterion: the higher the IRR, the more desirable the investment
- However, a potential investment is not worth considering unless its IRR is greater than the prevailing interest rate!
- IRR used for long-time investments, where investments cycle is to be repeated many times
- NPV comparison is to be used for one-time opportunities; preferred by theorists

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Descriptive Statistics

- Historical data collected in order to enable modeling of economic mechanisms and validation of models
- Data collected are stochastic – chance involved!
- Continuous and discrete data measurement
  - max. daily interest rate is dicrete
  - temperature is continuous
- Continuous and discrete data values
  - discrete data result from counting
  - continuous data can take any real value (e.g. interest rate)
- Example: Downloading the data, importing them into a spreadsheet, applying FREQUENCY function, plotting data