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Cholesky Decomposition

- Covariance matrix V is positive definite
- This allows for the following decomposition of V: $V = L\,U$, where L is a lower triangular matrix, and U is an upper triangular matrix

$$V = \begin{bmatrix} Var(R_1) & COVAR(R_1,R_2) & COVAR(R_1,R_3) \\ COVAR(R_2,R_1) & Var(R_2) & COVAR(R_2,R_3) \\ COVAR(R_3,R_1) & COVAR(R_3,R_2) & Var(R_3) \end{bmatrix} =$$

$$= \begin{bmatrix} a & 0 & 0 \\ x & b & 0 \\ y & z & c \end{bmatrix} \begin{bmatrix} a & x & y \\ 0 & b & z \\ 0 & 0 & c \end{bmatrix} =$$

$$= L\,U$$

- Example: For a $V = \begin{bmatrix} 0.0144 & 0.0199 & 0.0148 \\ 0.0199 & 0.0529 & 0.0231 \\ 0.0148 & 0.0231 & 0.0225 \end{bmatrix}$

we have

$$LU = \begin{bmatrix} a^2 & ax & ay \\ ax & x^2+b^2 & xy+bz \\ ay & xy+bz & y^2+z^2+c^2 \end{bmatrix},$$ so

Cholesky Decomposition

$a^2 = 0.0144 \Rightarrow a = 0.12$

$ax = 0.0199 \Rightarrow x = \frac{0.0199}{a} = 0.1658\dot{3}$

$ay = 0.0148 \Rightarrow y = \frac{0.0148}{a} = 0.12\dot{3}$

$x^2 + b^2 = 0.0529 \Rightarrow b = \sqrt{0.0529 - x^2} = 0.1593716\ldots$

$xy + bz = 0.0231 \Rightarrow z = \frac{0.0231 - xy}{b} = 0.0166104\ldots$

$y^2 + z^2 + c^2 = 0.0225 \Rightarrow c = \sqrt{0.0225 - y^2 - z^2} = 0.0837436\ldots$

$$L = \begin{bmatrix} 0.12 & 0 & 0 \\ 0.1658 & 0.1594 & 0 \\ 0.1233 & 0.0166 & 0.0837 \end{bmatrix}$$

– Mathematical theorem states that, if $V$ is decomposed into $V = LU$, and if a vector $n = \begin{bmatrix} n_1 \\ \vdots \\ n_N \end{bmatrix}$ is composed out of normaly distributed random variables, then $Ln$ has a variance matrix $V$

Cholesky decomposition

- This provides necessary instruments for simulating assets behaviors

- Example in Excel, incl. plots
(Normal distribution: NORMSINV(RAND()) )