Weak measurements in open quantum systems and in nonlinear optics

Lorena Ballesteros Ferraz[1], Julien Fagnart[1], Julien Bouchat[1], Riccardo Muolo[2], Timoteo Carletti[2], John Martin[3], and Yves Caudano[1]

[1]*Research Unit Lasers and Spectroscopies (UR-LLS), Department of Physics,*
[1,2]*Namur Institute for Complex Systems (naXys) &* [1]*Namur Institute for Structured Matter (NISM),*
[1,2]*University of Namur, Rue de Bruxelles 61, 5000 Namur, Belgium*
[3]*Institut de Physique Nucléaire, Atomique et de Spectroscopie, CESAM, University of Liège, Belgium*

A quantum weak measurement consists of (1) a preparation of a quantum state, (2) a weak interaction of the quantum system with a meter to probe an observable without disturbing much the system state, (3) a projective measurement of the system called post-selection that sets the system final state, and (4) a meter measurement conditional on successful post-selection. Observations depend then on a complex number called a weak value. Weak values are called anomalous when they do not correspond to any possible expectation value of the observable of interest. When the prepared and post-selected states are nearly orthogonal, very large anomalous weak values enable probing small parameters with increased sensitivity, a phenomenon called weak-value amplification.

We modeled weak measurements in the presence of dissipative dynamics of the probed quantum system [1] using Lindlad's equation. The dissipation and dephasing incurred by an atom after its interaction with a cavity can be probed by measuring the cavity field in a post-selected weak measurement. We connect the weak measurement outcomes, the weak value, and the atom properties as a function of the post-selection time. Weak values are affected by the dissipative dynamics. Anomalous weak values and amplification are generally not preserved in the steady state. Anomalous weak values enable differentiating between Markovian and non-Markovian dynamics.

We also reveal the usefulness of amplified weak measurements in the context of nonlinear optics to probe the spatial and angular Goos-Hänchen shifts occurring for paraxial Gaussian beams emitted by a surface nonlinear optical process, such as second-harmonic generation (SHG). The Goos-Hänchen shifts are tiny deviations, in the incidence plane, from the laws of geometrical optics, observed when applying these laws to the central axes of incident and reflected light beams. The axis of the reflected beam does not generally emanate from the surface at the incidence point of the impinging beam (spatial shift) and the angle of the axis of the reflected beam does not generally match exactly the incidence angle (angular shift). We reflect as well on the analogy connecting these two optical shifts to the difficulty of defining a proper duration to quantum tunnelling.

Finally, using Henrici's departure from normality to quantify non-normality, we study the correlation between the operator non-normality and anomalousness in weak values [2] in their polar geometrical representation [3]. A non-normal operator does not commute with its Hermitian conjugate, a stronger requirement than non-Hermiticity. Its eigenvectors are not orthogonal, and its average values are not bound to the convex hull of its eigenvalues. We further evidence a very general connection between quantum fluctuations and non-normality.

[1] *Revisiting weak values through non-normality*, Lorena Ballesteros Ferraz, Riccardo Muolo, Yves Caudano, and Timoteo Carletti, J. Phys. A: Math. Theor. **56** (2023) 475303 (30pp). Link

[2] *On the relevance of weak measurements in dissipative quantum systems*, Lorena Ballesteros Ferraz, John Martin, and Yves Caudano, Quantum Sci. Technol. **9** (2024) 035029. Link

[3] *Geometrical interpretation of the argument of weak values of general observables in N-level quantum systems*, L. Ballesteros Ferraz, D. L. Lambert, and Y. Caudano Quantum Sci. Technol. **7** (2022) 045028. Link