Stability of breathers in nonlinear Schrödinger-type models

Breathers are spatially localized nonlinear waves, with energy oscillating periodically in time. One of the paradigmatic continuous-wave equations for studying breathers in conservative systems is the integrable nonlinear Schrödinger (NLS) equation, with applications in many branches of physics, e.g. nonlinear optics, fluid dynamics, plasma physics, condensates of interacting bosons,...

Breathers may also appear in damped-driven variants of the NLS equations, such as the Lugiato-Lefever equation (LLE) frequently used for studying nonlinear phenomena in optical cavities with dissipation.

Moreover, spatially discrete versions of these models are frequently used to model wave propagation in various types of lattices ("tight-binding approximation"), most notably for coupled optical waveguides and Bose-Einstein condensates in optical lattices, which also support breather-like oscillation modes under quite general conditions.

In this talk, we will give a brief general overview of properties of breathers in the above mentioned models. In particular we will focus on breather stability: while the analytically exact breather solutions to the integrable NLS equation are generally unstable, breathers in damped-driven and/or discrete systems can be stabilized in quite broad parameter regimes. As the latter systems are in general not analytically solvable, numerical Newton-type methods are used to obtain breather solutions to computer accuracy, allowing precise determination of their linear stability properties through numerical Floquet analysis, and identification of the various bifurcations (Hopf, period-doubling,..) that limit regimes of existence and stability. We also show how strongly localized and highly discrete breathers in the lattice version of the LLE may smoothly transition into continuous LLE breathers under parameter variation, thus putting specific results obtained earlier in different contexts into a common framework. If time allows, some preliminary results on breathers in multicomponent LLE may also be discussed.

- M. Johansson, V.E. Lobanov and D.V. Skryabin, Phys. Rev. Research 1, 033196 (2019)