



The other cosmological parameter: wanderings, selection effects, and biases in a struggle to measure q_0

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Abstract The tumultuous history of measurements of the deceleration parameter q_0 , one of a few key cosmological parameters, yields historical, theoretical, and epistemological reasons for studying it. A wide variety of observational methods and techniques, combined with evolutionary, astrophysical, cosmological, and observational selection effects, as well as various, and often contradictory assumptions behind various models, led to widely differing values of q_0 (from -2 to $+36$!) over a span of half a century. Moreover, there was an inevitable epistemic trade-off between observational sensitivity and statistical wealth of data. While there certainly was a diversity of theoretical predictions for q_0 among various cosmological models, our analysis indicates that the main historical cause of such a large and confusing scatter of empirical values was the persistent failure to appreciate the scope and tempo of evolutionary change in various astronomical sources. We survey and sort out this historical context, providing some preliminary general lessons for ongoing and future endeavors of similar complexity.

1 The changing values of q_0

Ever since Allan Sandage characterized observational cosmology as the search for two numbers, the present expansion rate, or Hubble's "constant" H_0 , and the present deceleration parameter q_0 , (Sandage 1970), there has been a persistent interest in the methods and approaches to finding out the values of these numbers. Historically, the two stories have been extremely different. While H_0 and multiple controversies surrounding it have been in the spotlight of many studies (Longair 1971; Trimble 1996; Freedman & Madore 2010; Jackson 2015), there has been comparatively little attention to measurements of the deceleration parameter q_0 . At most, it has deserved a (sub)section in general reviews of observational cosmology (e.g., Kragh 2007; Kragh & Longair 2019; Coles & Lucchin 1995) or a chapter in graduate cosmology textbooks (e.g., Peebles 1993, ch. 13; Coles & Lucchin 1995). The goal of the paper is to start rectifying this situation, and to elucidate the reasons for this difference.

As far as the historical aspect of our subject matter is concerned, we survey the observational results on q_0 belonging to the 1952–2010 interval. Formally, we may bookend this 6-decade period by Walter Baade's revision of the extragalactic distance scale in the early 1950s on one side (Baade 1952) and the 2010 results of the Supernova Legacy Survey (Guy et al. 2010), on the other; the latter cemented the discovery of the accelerating universe by two supernova collaborations in 1998 (Perlmutter et al. 1998). The Nobel Prize in Physics for 2011 quickly followed, awarded to Saul Perlmutter, Brian P. Schmidt, and Adam G. Riess for "the discovery of the accelerating expansion of the Universe through observations of distant supernovae."¹ It is reasonable to speculate, until the Nobel Foundation archives are opened, that it was exactly the outcome of the Supernova Legacy Survey that prompted the decision. In brief, and as mentioned in the background material published by the Royal Swedish Academy of Sciences, the Nobel *was* given for establishing that $q_0 < 0$. Together with solidifying evidence in the 2005–2010 period that the true value is $q_0 \approx -0.6$, this provides us with a well-defined end of the controversy—as good an ending as can ever be the case with scientific controversies (cf. Kragh 1996).

Much has happened between those bookends, however, including the entire mini-revolution associated with the discovery and observations of the Cosmic Microwave Background (CMB) radiation which brought us into the age

¹ <https://www.nobelprize.org/prizes/physics/2011/summary/>, last accessed October 9, 2025. The advanced scientific background is given at <https://www.nobelprize.org/uploads/2018/06/advanced-physicsprize2011.pdf> (last accessed October 9, 2025).

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Table 1 Observational determinations of q_0 from Hubble to this day (a non-exhaustive list). Where no uncertainty Δq_0 is listed, the authors did not provide it, usually because they cite just the upper or lower limit

No.	q_0	Δq_0	Study	Method	Comment
1	2.6	0.8	Humason, Mayall & Sandage (1956)	Redshift–magnitude relation for a large sample of both field and cluster galaxies	An extremely influential, “Bible-like” catalog of galaxies, inspired by Hubble
2	1	0.5	Baum (1957)	Redshift–magnitude relation for 7 galaxy clusters	
3	1	0.5	Sandage (1961)	Redshift–magnitude relation	
4	– 0.06 – 1.7		Solheim (1966)	Redshift–magnitude relation for 20 galaxies and 15 QSOs	Results correspond to i. galaxies in clusters; and ii. QSOs
5	0.09	0.09	Zotov & Davidson (1970)	Redshift–magnitude relation for 147 QSOs	
6	1.5	0.4	Peach (1970)	Redshift–magnitude relation for brightest cluster galaxies	
7	1.03	+ 0.43 – 0.26	Sandage (1971)	Brightest galaxies in clusters	“At present, none of this must be considered very seriously.” (p. 618)
8	0.3	0.2	Baum (1972)	Angular size–redshift relation for individual galaxies in clusters	The first use of the angular size test, suggested by Hoyle (1959)
9	0.96	≈ 0.4	Sandage (1972)	First-ranked galaxies in clusters	
10	0.03	0.01	Gott et al. (1974)	Various local observations	
11	1	~ 1	Austin & Peach (1974)	Galaxy counts in rich clusters	“[T]his scheme leads with our data to a value of q_0 slightly greater than + 1 with an uncertainty of about ± 1 , but we place little weight on this conclusion.” (p. 455)
12	1.07	0.88	Kollerstrom & McVittie (1975)	Galaxy clusters	The “limiting” model among several; reinterpretation of the data of Sandage (1961, 1971)
13	0.31 – 0.15	0.68 0.57	Gunn & Oke (1975)	Faint galaxies in clusters + 3C 295 radio galaxy	The first result includes 3C 295, the second does not
14	1.38		Fang et al. (1976)	Redshift–magnitude relation for radio-loud QSOs	
15	– 0.8	0.2	Hickson (1977)	Angular size–redshift relation for clusters	Corrected for the <i>then known</i> evolutionary effects – and strangely prescient
16	~ 1		Davidsen, Hartig, & Fastie (1977)	The spectrum of 3C 273	3C 273 is not only located behind the Virgo Cluster, making some of its spectral lines confusion-limited, but it seems to be intrinsically atypical

Table 1 (continued)

No.	q_0	Δq_0	Study	Method	Comment
17	0.25	0.5	Bruzual & Spinrad (1978)	Angular size–redshift relation for clusters	“The evolutionary correction on q_0 is unknown at present, but may be large.” (p. 7)
18	≈ 2		Baldwin et al. (1978)	Emission spectra of QSOs	Claims that low q_0 values (≈ 0) can be decisively rejected
19	1.6	0.4	Kristian, Sandage & Westphal (1978)	Brightest galaxies in clusters	Cf. items #7, 9, 25
20	0.02	0.01	Yahil, Sandage & Tammann (1979)	Virgocentric infall	Method sensitive to local peculiar velocities (poorly constrained at the time)
21	~ 5		Turner (1979)	Radio luminosity function for QSOs	A kind of <i>reductio ad absurdum</i> of the no-evolution approach
22	$8.4 \times 10^{-4} < q_0 < 0.2$		Bode & Evans (1979)	Transparency of intergalactic medium	
23	1.86	1.36	Kiang & Cheng (1982)	Redshift–magnitude diagram for QSOs	
24	36 1.6 5.2 0.7	5 0.1 0.4 0.1	Hu (1982)	Redshift–magnitude diagram for QSOs	Results correspond to QSOs: i. with resolved radio components; ii. with strong interplanetary scintillation; iii. light variation no bigger than 1/2 magnitude; and iv. resolved by VLBI
25	-1.2	0.6	Bigot, Fliche & Triay (1988)	Brightest galaxies of Abell clusters	Using a novel statistical method of null correlation
26	> -2.3		Gott, Park & Lee (1989)	Gravitational lensing	The vague limit follows from curves for optical depth to lensing as a function of the comoving distance to the source
27	0.05		Guiderdoni & Rocca-Volmerange (1990)	Faint galaxy counts	
28	≈ 0.5		Kellermann (1993)	Angular size–redshift relation for compact radio sources	
29	2.6	$+2.1$ -2.2	Stepanas & Saha (1995)	Angular size–redshift relation for VLBI compact radio sources	“[W]hile the data are consistent with $q_0 = \frac{1}{2}$. . . , they do not at present usefully constrain q_0 .” (p. L15)
30	0.8	0.65	Perlmutter et al. (1996)	SN 1994G	“The data do, however, indicate that a decelerating $q_0 > 0$ is a better fit than an accelerating $q_0 < 0$ universe.” (p. 27) SN 1994G was discovered at what at the time was the record distance, but it was seemingly atypical

Table 1 (continued)

No.	q_0	Δq_0	Study	Method	Comment
31	0.85	0.29	Pen (1997)	Angular size distance to clusters	Cluster sizes exhibit much inherent scatter and evolve much more rapidly than hitherto assumed
32	0.10	+ 0.65 – ?	Schuecker et al. (1998)	Redshift-volume test for a large sample of $z < 0.2$ galaxies	
33	0.07	+ 0.60 – 0.54	Rines et al. (1999); Sasaki (1996)	Gas mass fraction in nearby versus distant clusters	95% confidence result
34	≈ 0		Wiik and Valtaoja (2001)	VLBI observations of shocks in AGN jets	Another “standard ruler” attempt
35	1.142		Xie et al. (2001)	Redshift–magnitude relation for metal-rich QSOs	Perhaps the last attempt to use QSOs for measuring q_0
36	– 0.35	0.15	Daly & Djorgovski (2004)	Coordinate distances to SNe and radiogalaxies	
37	– 1.00	+ 1.26 – 1.23	Liang & Zhang (2005)	Gamma-ray burst luminosities	An original method which did give best-fit $q_0 < 0$, albeit with huge uncertainty. It turns out that GRBs have large intrinsic scatter of luminosities
38	– 0.58	0.17	Gong & Wang (2006)	Parametrization of data gathered from multiple SNe datasets	
39	– 0.61	0.11	Astier et al. (2006)	Supernova Legacy Survey (1 year)	
40	– 0.58	0.23	Xu & Liu (2008)	182 Type Ia SNe + HST data on differential ages of galaxies	
41	– 0.68	0.15	Guy et al. (2010)	Supernova Legacy Survey (3 years)	The crowning result of the series of observations showing that sets of SNe not only persistently give negative results for q_0 , but also converge on the value of – 0.6
42	< 0		Schrabback et al. (2010)	Weak gravitational lensing	96% confidence result, useful as an independent confirmation of accelerated expansion, although weak lensing is strongly confusion-limited
43	– 0.57	0.04	Neben & Turner (2013)	Constitution set of Type Ia SNe	“The end of history”: summarizes the New Standard Model view of q_0

of “precision cosmology.” The road, however, was somewhat windy and complicated even there (Peebles, Page and Partridge 2009; Perović & Ćirković 2024). We should also keep in mind that CMB in the post-COBE era—and within the framework of its orthodox interpretation as the redshifted remnant of the primordial optically thick fireball—enabled testing of cosmological parameters which is completely independent from properties of discrete sources. It is only in conjunction with CMB data that the supernova surveys provided foundation for building of the “New Standard,” post-1998 Λ CDM (= dark energy λ + **C**old **D**ark **M**atter) model.

The measured value of q_0 has evolved significantly over time, reflecting advances in observational techniques and theoretical understanding (Table 1). Early studies (1950–1970s) often reported positive values, typically around $q_0 \approx 1$, based on redshift–magnitude relations for galaxies and clusters. These results aligned with a decelerating universe dominated by matter. However, some outliers like Solheim (1966) and Hickson (1977) suggested negative values, hinting at possible acceleration or at least large methodological uncertainties. In the transitional period (1970–1990s), estimates became more varied, ranging from $q_0 \approx -1$ to $q_0 \approx 2$, as new methods (angular size–redshift relations, QSO (quasi-stellar objects) studies, gravitational lensing) and more precise evolutionary corrections were introduced. In 1989, Yoshii and Takahara (1989) stated that most measurements suggest $q_0 < 0.1$, while theoretical expectations converge on $q_0 = 0.5$. The modern consensus (late 1990s–present) was reached at the turn of the twenty-first century, which marked a pivotal shift. Supernova (SN Ia) studies (e.g., Perlmutter et al. 1996, 1999; Astier et al. 2006; Betoule et al. 2014) consistently favored negative values close to the modern-day value ($q_0 \approx -0.6$). Independent methods like gamma-ray bursts (Liang & Zhang 2005) and weak lensing (Schraback et al. 2010) confirmed this trend, even if vaguely, while still plagued by large uncertainties. By the 2010s, q_0 settled near -0.6 with small uncertainties (Neben & Turner 2013). This established the “New Standard Model” of physical cosmology—a universe undergoing accelerated expansion due to dark energy, with matter playing a secondary role at later epochs. The New Standard Model largely took role of the “default” which has hitherto belonged to the Einstein-de Sitter (henceforth E-dS) model, albeit with much stronger empirical support.² Table 1 gives a list of some highlights of values and parameters obtained in observations over these periods (see also theoretical summary in Appendix 2).

2 Meandering development of observational methods and techniques

Wide-ranging values of the deceleration parameter q_0 emerged within a wide variety of observational methods and techniques (see Appendix 2). Until a fairly reliable determination of the absolute luminosities of galaxies, primarily by estimating the luminosity of Cepheid variable stars, even rough observational estimates of q_0 were not possible. Also, the evolutionary trajectory of elliptical galaxies was not sufficiently known until the mid-1960s, which was crucial for estimates of their absolute luminosity and hence their distance. Evolutionary correction is crucial; one has to know the absolute magnitude characterizing a galaxy in a particular stage of its evolution, in order to estimate how far away it is based on its apparent magnitude. Also, it is a crucial lucky fact (Sandage 1968, 95) that the variation in the absolute luminosity of galaxy clusters is smaller than deviations from linearity in the redshift law. If the opposite were true, we could not determine q_0 observationally using clusters or cluster galaxies.

The redshift–magnitude method was first proposed by Hoyle and Sandage in 1956. Any relationship of density, curvature, and radius of the universe makes sense in practical terms only if q_0 can be determined observationally. The practical idea was that the changing rate of H could be determined by measuring “nebulae” at different distances, measuring the distance and determining velocity via redshift across the epochs. Thus, it was expected that the bolometric determination of the magnitude and the measured value of redshift would enable such data across epochs. The key was that the rate of change in absolute magnitude is in principle “amenable to observation” (Hoyle and Sandage 1956, 302), the complications of the evolutionary curve of apparent magnitude changes and other difficulties aside.

In general, in the late 1960s and early 1970s the redshift–magnitude values were reliable enough only for setting upper and lower bounds for the q_0 value. This was made quite apparent by Peach (1970) who analyzed photometric

² In retrospect, the historical fact that E-dS model was so widely accepted in roughly 1965–1998 period does seem surprising—and has not been adequately treated by historians of cosmology so far. The empirical evidence for it has always been thin: neither the quantity of detectable matter nor the age of the oldest stars were ever adequately accounted for by the model, not to mention more sophisticated tests such as the measurements of q_0 based on standard candles or standard rulers discussed here. It is difficult to avoid the impression that it is only the *analytic ease* of calculating things in the E-dS—and perhaps the controversial “mathematical elegance”—which kept it as a default for such a long time (O’Raifeartaigh et al. 2015). Challenges to this orthodoxy, even when coming from distinguished experts (Beatrice Tinsley and coworkers in 1970s, Steven Weinberg in 1980s), were mostly shrugged off *until essentially the last moment* when the supernovae and the CMB data showed that the accelerating expansion was real.

data by Sandage and his collaborators and offered a range of possible values and error estimates for approximately 40–50 data points. The values for this rather meager (in hindsight) dataset ranged from 0 to 1.68.

The first substantial attempt at determining q_0 based on redshift–magnitude was a compilation put together by Sandage in 1972. He analyzed various datasets estimating the value based on 41 and 84 clusters. A small residual value of Hubble expansion corresponded to q_0 of + 1, reduced to 0 due to evolutionary correction estimates, but overall Sandage warned (Sandage 1972, 23) that the results were preliminary given the ignorance on how well Friedmann models describe the actual universe. He was fully aware of various assumptions and selection effects at play which he discussed extensively. Nevertheless, this was the first fairly comprehensive data analysis, establishing the template that was subsequently followed and expanded.

In the early 1970s, the challenges emerged in determining evolutionary corrections for cosmological measurements. Thus, Sandage (1971) highlighted how difficult it was to calculate reliable evolutionary corrections given the limited data and varying source populations, concluding that available estimates were unreliable beyond redshift $z = 0.2$ without more observations. Tinsley (1972) cautioned, estimating that evolutionary corrections to the deceleration parameter q_0 needed at least an additional 0.5 magnitude, based on assumptions about star formation and mass distribution. And together with Brown, she further warned that magnitude–galaxy count methods might detect source evolution rather than true cosmological effects (Brown & Tinsley 1974).

The first fairly comprehensive estimate of q_0 and the density parameter based on redshift–magnitude observations of twenty cluster galaxies, corrected for evolutionary effects, was published by Solheim in 1966. The paper also presented data on fifteen QSOs and the result for q_0 was -1.7 , which agreed with the galactic observations with evolutionary correction. This was one of the earliest surprising results indicating a substantial negative value.

In 1970, based on 147 QSOs, Zotov and Davidson (1970) determined a surprisingly low value of q_0 (between 0 and 0.18) based on redshift–magnitude analysis. This range was estimated accounting for the possibility that the value of redshift value could be affected by intrinsic evolution of the objects.

In 1962, Harvard astrophysicist David Layzer suggested the basics of determining q_0 by measuring *the angular diameter of galaxy clusters* across redshifts known at the time and compared the models based on the method with already existing, and admittedly deficient, data. Strangely enough, Layzer’s intriguing study has been completely ignored.³ It took more than a decade before Austin and Peach (1974) presented estimates of q_0 close to + 1, based on the same parameter. The data concerning nine clusters of intermediate redshifts were analyzed and a definition of cluster diameter offered. In a big shift in terms of the constant’s value based on the identical method, Hickson’s (1977) best estimate of a $q_0 = -0.8 \pm 0.2$ resulted from the analysis of angular size–redshift measurements of some 90 clusters and tallying of data with various models. It turned out to be the most prescient of early studies—and going quite against the prejudice of the day.

With the more precise measurements of the CMB after its 1965 discovery, the idea of determining q_0 based on a comparison of *distortions in the CMB due to the Sunyaev–Zeldovich (SZ) effect* via comparing two clusters at different redshifts came to fruition in the late 1970s. In 1979, Fabbri and collaborators (Fabbri et al. 1979) proposed an observation based on the SZ effect as well. The authors proposed to use a two-band survey of anisotropies across large redshift clusters across the skies. The angular size of clusters depends on evolution, but that is something that can be dealt with empirically with a standard-candle approach in the case of CMB–SZ interaction.

Following various, ever-more precise measurements of the CMB anisotropies (quadrupole anisotropies, later determined to be the result of the motion of the Local group, the Solar System, and other local effects), squaring the models with the measurement results available thus far Raine and Thomas (1981) showed that small-scale anisotropies were fixed to small values of q_0 , and that only large-scale anisotropies would result in a substantial increase of its value. A small q_0 based on small-scale anisotropies became, ipso facto, an argument for the homogeneity of the universe—an inversion of the usual methodology applied in physical cosmology since Einstein.

In 1988, Krolik and Raymond (1988) suggested a variation of the measurement technique of X-ray sources that stand out in terms of the magnitude of the CMB due to the SZ effect. They suggested measuring such sources in front of quasars bright in the X-ray region of the spectrum. This variation of the technique was expected to have an advantage of low noise in measuring the key parameter—normally the tail of the CMB spectrum. The authors offered a preliminary analysis of particular spectral lines relevant for such analysis, which they deemed viable in the near future, as well as potential sources—lined up in the Abell-class clusters.

As far as the development of observational techniques and apparatus goes, the photographic measurements of galaxies’ apparent magnitudes increased rapidly after the initial ones of a few sources in the early 1950s, while preliminary photoelectric measurements of the same period quickly turned into automatized data-processing in the 1960s. Spectral analysis of colors (spectrum bands), as well as photometric techniques, improved corrections in the analysis of spectra and provided more refined tallying of observations with predicted evolutionary stages (and thus estimates of absolute magnitudes). A breakthrough came with spectrophotometry (Baum 1957, 1972) of redshift that bypassed K corrections (corrections due to regular spectral change of moving objects). The theoretical estimates of color change with redshift could be compared to the observational results, and thus, various models could be tested. Bolometric measurements have been crucial, in particular, in thermalization measurements

³ By the time of this writing (March 2026), it has only 6 citations (excluding autocitations) as per Google Scholar database.

related to the CMB attenuation (SZ effect) in hot gas clouds. The galaxy and cluster manual counts done on the photographic plates at only a couple of observatories (Palomar) have steadily improved and diversified in terms of the telescopic instruments and their parameters. And in the 1980s we see the appearance of papers pursuing cross-comparisons of techniques, the use of joint parameters from different techniques in statistical analysis, and long-lasting calibrations over time. The multi-instrument photometric data were analyzed by Sandage (1972). The Palomar and Mount Wilson teams used photomultipliers in six observational sessions, collecting over 80 cluster data points. This was a milestone in terms of the use of multiple techniques and quantity of data.

The details and the first estimates of the techniques of counting galaxies within clusters were presented by Layzer (1962). The technique involves the distance—based on both magnitude and redshift—and the radius of the cluster, via the count of galaxies brighter than a set value. The brightest galaxy served as the center around which other galaxies were counted, and the redshift was determined for that central galaxy alone (Bruzual and Spinrad 1978). The 1984 attempt at standardization by Shanks and coauthors (Shanks et al. 1984) analyzed wide-angle photographic counts on 4-m telescope photographs, at deep limits where selection effects were more likely to become obvious. They looked at two techniques of determining magnitudes: determining total magnitude and isophotal magnitude.

Starting a line of the evidence that will turn out to be crucial, in 1977 Wagoner argued for *supernovas as standard candles* in measurements of q_0 , as they uniformly have a straightforward evolutionary function. There is a simple thermal continuum over a short (40-day) initial burst across supernovas—the temperature and luminosity can be easily and accurately measured given the black-body spectrum of the exploded star. As there is a simple spectrum change for every z , the luminosity-redshift determination should be straightforward. In fact, the basic technique was quite old, dating back to the 1920s and Baade's work, and it was tried on various individual supernovas.

Yet, early hopes that supernovas would serve as cleaner standard candles than galaxies were soon tempered. Despite drastically reduced evolutionary correction errors, the selection effects from peculiar and rotational galactic velocities complicated redshift estimates and q_0 determinations, as Phillipps (1983) discussed.

Thus, creating effective techniques and especially accumulating sufficient data for estimates of q_0 had to wait for the 1970s, and then the late 1990s (Newberg 1992) and early 2000s, respectively. The discovery and explanation of the SZ effect and sources affected by it are new in comparison. A general technique for their use as standard candles in q_0 measurements was explicated in the mid-1970s.

Finally, gravitational lensing of galaxies and QSOs was assessed with respect to various cosmological models and values of q_0 by Gott, Park and Lee (1989), with the resulting estimate of $q_0 > -2.3$ (See Appendix 3).

3 The challenges of standardization

A wide-ranging diversity of observational methods certainly contributed to the wide-ranging values of the parameter over six decades. The emerging standardization across instruments and parameters started emerging fairly late. The extensive data on the southern sky followed (Sandage & Tammann 1975), with skepticism regarding the evolutionary correction for spiral galaxies, a substantial portion of the datasets from two southern observatories (Mount Stromlo and Siding Spring). The best estimate of q_0 was equal to or less than 0.02. Yet the same year Kollerstrom and McVittie (1975) argued that q_0 less than zero is the only unviable option in correlating various values of the cosmological constant and existing data that Sandage gathered and discussed over the years, and that the most acceptable models were those having $q_0 = 1$ (!).

As far as the observational selection challenges from instruments and calibrations go, the K term (or K-correction, see Sect. 4.4 for the details) was a notorious case. It plagued all observational projects based on magnitudes of sources; one may think that observations relying on counts would fare better. In fact, the determination of q_0 through galaxy cluster counts was plagued by multiple observational selection effects we discuss below, all contributing to skepticism toward physical cosmology until the early 1990s.

Sandage (1971) and especially Whitford (1971) standardized aperture effect and K-dimming fairly early on. And Ellis, Sievers, and Perry (1984) created the so-called observational map for cosmological observations of galaxies in 1984. Enough data had accumulated by that time, and various selection effects related to the observational process and instrumentation had been categorized and fairly well understood, if not addressed adequately, so that a usable summary standardization was a plausible project. The idea was to establish a relationship between the physical properties of sources and the parameters of images, taking into account various distortions across observational contexts. The resulting cosmological functions depend on adequately handling selection effects in the transfer of source functions and parameters to detector functions and parameters. Two key dependencies in estimating how exactly an object plane projects onto a telescopic image were the dependence between surface brightness of the observed object and the point spread on the image (whether distinct points on the image due to incoming photons represent distinct sources).

Similarly, the same year, Shanks, Stevenson, and Fong (1984) attempted a summary standardization for galaxy counts in the cosmological context. They listed all observational parameters and calibrations for wide-angle counts and analyzed their differences across instruments and various diverging datasets they produced.

And in a comprehensive review of observational methods and motivation for observing H_0 and q_0 , Sandage (1968) provided a diagram of the dependence of *spectrum colors on redshift* for 31 brightest galaxies in clusters. This was quite a big effort for the time and the contemporary inefficient data analysis methods, and it should have helped researchers calibrate results for their own samples by removing the *dependence of K-correction on redshift*. However, the reality proved messier than expected, for multiple reasons. For instance, observers used the spectrum of the nearby compact elliptical galaxy M32 as a prototypical zero-redshift elliptical galaxy spectrum. Unfortunately, M32 is in fact quite *atypical*—to the degree that a currently popular theory of its origin suggests that today’s M32 is essentially just the bulge of a previously existing *spiral* progenitor object M32p, whose bigger part was stripped by and absorbed by its huge neighbor, M31, as late as 2 Gyr ago (D’Souza & Bell 2018)!

Even more important problem with Sandage (1968) color-redshift diagram is that it extended “only” to $z = 0.2$ (we are talking 1960s, recall, the time at which a large fraction of astronomers disbelieved QSO redshifts as being cosmological in origin), some of the most distant being actually detected through *radio* surveys, a strong indication of atypicality. Modern understanding of the difference between spectra of star-forming versus passive sequence galaxies, not to mention the “green valley” separating them, came only about 2000 (see, for instance, the review of Salim 2014). Taken together, these circumstances effectively condemned the redshift–magnitude and redshift–color methods to massive confusion until the turn of the century.

Linder (1988) offered both methodological and numerical standardization of variations of lookback time, comoving volume, and angular diameter both in a smooth and clumpy universe—with the change of redshift.

Crucially, various approaches differed in terms of very general cosmological, and more specific astrophysical assumptions often centered around the treatment of the cosmological constant (see Appendix 2). Thus, observational standardization alone could enable convergence. Reevaluation of the cosmological constant started explicitly in the early 1970s—Peach (1970) acknowledge it was “unfashionable” but inevitable in probing various assumptions. In fact, already in 1962 Layzer sorted out a number of different and often contradictory assumptions figuring across models that agreed, or were supposed to be tested, with different lines of evidence. Finally, an early example of a rather mundane but important challenge, where an old estimate simply was carried over uncritically to future research—a fairly widespread effect—was pointed out by Sandage (1972, 8).

Thus, the cosmologists identified and grappled with these challenges throughout. So far, we have only touched on some of them while surveying the historical development of the determination of q_0 . Yet, the diversity of observational methods in combination with various selection effects, and various, often contradictory, assumptions—the magnitude of which led to widely differing values of the parameters—merits a closer analysis.

4 Mitigating selection effects

Generally speaking, selection effects discussed below stem from observational limits in time and in space. The parameters figuring in them are not fundamental theoretical assumptions concerning q_0 but certain features (often incidental) of the observed sources, which directly affect the observational results. They result from inevitable partial collecting of observational data, or systematically skewed data, that lead to inadequate inferences concerning properties of sources and physical processes that are used to estimate q_0 . The selection effects are classified based on whether they are tied to direct observational parameters or based on features of astrophysical and cosmological models indirectly skewing the observations.

4.1 Evolutionary

The evolution of galaxies is a major factor in determining q_0 via the redshift–magnitude method. The brightness of galaxies, and thus their distance, will depend on how young/old they are since they evolve, and since the further we look into the universe the younger galaxies we see (on average). This is why determining the precise rate and stages of their evolution is crucial for inferring q_0 . This has been true especially of elliptical galaxies as they exhibit the simplest evolutionary changes, and are dominant in, and especially around the centers of galaxy clusters, since they are often first-ranked (brightest) galaxies in clusters (the items #7, #9, #19, and #25 in Table 1).

Sandage (1971) fully explicated how daunting the task is of coming up with the evolutionary correction based on all the measurements performed up to that point, given the extent to which the number of sources and epochs to which they belong will substantially but reticently affect the estimates. He concluded that diverging estimates cannot be taken seriously at all before many more objects are observed beyond $z = 0.2$ (Sandage 1971, 618).

Another strong cautionary note by Tinsley (1972) on the evolutionary correction set a milestone of sorts. She concluded, albeit as a rough approximation, that the evolutionary correction to q_0 cannot be lower than an additional 0.5 to the value measured via redshift–magnitude techniques. She based this conclusion on her

examination of two seemingly simple but in fact problematic assumptions; one concerning the star formation across epochs (only the initial “burst of duration” of the stars matters, compared to the present epoch), and another supposing a single power-law initial mass function for all stars at all epochs.

In a similar skeptical vein, the worry about the magnitude/galaxy counts method was voiced again by Tinsley and her coauthor (Brown and Tinsley 1974). The method, they argued, may be more sensitive to the evolution of the sources than to the cosmological effect. This was quite obviously true of radio sources at the time but it also applied to the optical region of the spectrum. To make things more complicated, spiral galaxies were expected to dominate the counts, yet elliptical galaxies were the ones expected to serve as standard candles due to a more stable evolutionary trajectory. Thus, knowledge of evolutionary functions for all types of galaxies was needed, and this was especially irritating because of the lack of clarity as to which morphological types dominate at fainter magnitudes. On the top of it, hypothetical intergalactic dust was suggested to affect the values of luminosity as per analogy with the extinction in the Milky Way disk.

Tammann (1974) summarized all of the mentioned questionable assumptions to the effect of overall skepticism about the galaxy magnitude-based methods without really deep knowledge of luminosity evolution in galaxies, concluding that early indications of the scale of the evolutionary correction show that the q_0 value will be pushed below the critical value of 0.5.

Evolutionary correction was an issue in counting clusters as well (Bruzual and Spinrad 1978, 6). The distance of clusters affected the counts, as closer ones with particular evolutionary corrections were more strongly represented due to easier access. For very faint objects, cluster identification became confusion-limited. Potentially the most extreme version of the effect was that if the clusters were the result of early inhomogeneity, then the early ones would collapse more quickly, and would be missing altogether in the observational data.

Yoshi and Takahara (1989) compared the scale of the theoretical evolutionary assumption on the one hand, and the selection effect due to the dominance of galaxy observations of certain magnitudes on the other. They concluded that the assumption was much less problematic than the selection effect, and recommended thorough observations at magnitudes of 23 and 24. They assessed the analysis of a single type of galaxies across q_0 values via computer simulations. Similarly, Guiderdoni and Rocca-Volmerange (1990) analyzed faint galaxies and concluded that, based on their comprehensive dataset, it was very hard to maintain the 0.5 value of q_0 favored by theoretical models (read: E-dS) at the time.

The initial enthusiasm for QSOs as standard candles for an early epoch was dampened quickly as it became clear that one would have to answer why they are distributed with an early and strong redshift peak. One way of addressing the question was to tally the redshift distribution of the QSOs with the expansion (i.e., the older they were the less redshifted) and deceleration. Petrosian (1970) used the existing data to try and discern whether the peak in distribution at about $z \approx 2.3$ and subsequent steep fall was due to cosmological or astrophysical reasons (in particular, due to the intergalactic medium opacity in early epochs, following a prescient suggestion of Martin Rees that QSOs themselves are responsible for ionization of intergalactic hydrogen). The short life of QSOs and their short-lived luminosity in comparison with galaxies made it unlikely that the redshift would be substantially affected, but the latter possibility was not entirely discarded (Zotov and Davidson 1970). Turner (1979) tried to tally the distance–redshift relation for QSOs with their luminosity/evolutionary function. The joint optical + radio function of luminosity showed that there was no standard luminosity and that it was independent of density evolution. It became clear that QSOs may be more complicated than supernovas as standard candles for q_0 determination. The controversy only deepened later on. Kiang and Cheng (1982) wrote a statistical analysis paper, warning that, potentially, the more rigorous the procedure of determining luminosity based on magnitude, the greater the danger that the outcome would provide a substantial assumption regarding the evolution of QSOs buried in the method. To make things worse, computerized analysis only made such calculations—and thus potentially the scale of error—more efficient. They looked at data determining q_0 based on redshift–magnitude from the width and separation of analyzed spectral lines. Moreover, as Phillips and Ellis (1983) argued, data on QSOs’ absorption line redshifts had to be interpreted as a result of evolving dust clouds for any reasonable value of q_0 .

Understanding the evolution of radio sources is critical for estimating the relevance of the rate of the sources’ change in angular size over time. This is why measurements of the ratio between angular size, flux, and density of sources—as well as their number—are crucial for such estimates. Yet this has proved to be a tall task as well. The 1987 analysis by Kapahi and coauthors concluded that evolution of the luminosity function for radio sources agrees with measurements only if cosmological linear changes in angular size are assumed even for $q_0 = 0$, that is, a freely expanding universe with negligible gravitational deceleration (Kapahi et al. 1987). The results were strongly dependent on the selected frequencies, but even when that effect was taken into account, it seemed that the conclusion concerning the cosmological origin of the linear evolution in angular size was inevitable.

4.2 General astrophysical

Galactic light dimming due to the absorption of some light by objects outside of our Galaxy, as well as the rotation of the Galaxy that produces systematic blue or redshifts in spectra, has to be taken into account in q_0 estimates based on the properties of galaxies and galactic clusters.

Early on, the hope was that using supernovas as standard candles due to their observable, brief, and simple evolution, compared to galaxy evolution, would drastically reduce the errors in evolutionary corrections, and effectively reduce the key measurements just to the other observed properties. But the hope was quickly tempered. The straightforward method for determining q_0 via this method was affected by potential selection effects related to peculiar galactic velocities and rotational velocities that substantially impact the estimates of redshifts (Phillipps 1983). The cure for this selection effect was measurements at higher redshifts, but the overall taming of the effect required hundreds of samples, a tall and long-term task given the rarity of SNe compared to the number of galactic clusters.

Once the key selection effects were identified and the mitigating strategies started emerging, at the end of the 1980s Linder (1988) provided a general test of Friedmann models with respect to general astrophysical properties. He theoretically tested how general astrophysical properties of sources—various properties related to distance—influence redshift values, irrespective of whether the evolution of the source is known (i.e., irrespective of idiosyncrasies of the source). Linder's paper is also a methodological study that compares methods and methodological general rules for the analysis of these astrophysical properties in the cosmological context that avoid selection effects.

4.3 Cosmological scale

A tendency to select richer galaxy clusters at greater distances is the common selection effect (Scott effect) in galaxy counts and angular estimates of their numbers. A somewhat mitigating factor is that it becomes significant only at very great distances. Janis (1986) explored variation in angular size with the expansion of the universe.

4.4 Observational instruments and calibrations

The so-called K-correction or K term, occurs due to the fact that the redshift of distant galactic spectra also distorts their magnitude in the blue region of the spectrum, as it reflects higher luminosity at frequencies that are higher than they would be for closer objects. The level of precise correction directly influences the spectral techniques we listed above (see also Appendix 3), which concern evolutionary rate estimates and observations. In 1968, Sandage wrote that “the history of the K term is long, involved, and frustrating.” (Sandage 1968, 96). This is indeed a long-lasting purely observational selection effect that frustrated progress across methods of determining q_0 . Analysis via the same emission frequency pioneered by Baum bypasses this distortion, but the level of its effectiveness is dependent on the actual technique used.

The aperture effect has been discussed throughout the decades of the measurements as one of the main instrument-calibrating issues. The image provided by the telescope can sharpen a small segment of the field at the expense of the clarity of the rest of the image, or it can distribute the clarity across the visual field without the benefit of a clear detail. The edges of galaxies will fade into the background, but this parameter has to be standardized across instruments. The estimates will depend on the focal length. With the emergence of larger datasets, tens of observed clusters, a need for standardization across instruments emerged (Sandage 1972).

Whitford (1971) analyzed the intensity of blue lines in galactic spectra affected by the aperture effect and standardization issues. His analysis of K-dimming by color in the spectra resulted in the conclusion that the difference between real and apparent values of q_0 via redshift–magnitude observations is around 0.5 due to the effect. It was clear at that point that a combination of observational selection effects alone may produce substantial differences in the q_0 estimates.

Similarly, the distortions will vary from the center to the edges of the photographic plate as well, which has to be corrected, as the estimates of the velocity of recession will be affected. The first systematic study of this effect across instruments was undertaken by Gudehus (1972). He suggested a calibration technique of centering (because the image was least deviated at the center) by using multiple photographic plates, and warned that the effect may be substantially amplified by inadvertent selection of objects with fainter magnitudes at the edges, and suggested a solution. Also, the reddening of the spectrum in the northern hemisphere due to atmospheric dust depending on the elevation, or reddening due to interstellar clouds and absorption in the Milky Way (Sandage 1972, 7), has to be corrected precisely, as it affects redshift estimates.

The inability to distinguish between faint stars and galaxies within an observed cluster was noted by Austin and Peach (1974, 438). Also, Hickson (1977), who analyzed the largest sample to date of angular size–redshift measurements, of some 90 clusters, pointed out that much older clusters with QSOs must be included in future observations. The cluster sizes were fairly uniform when determined by two distinct methods, but the applicability

of these methods to distant clusters was in question. Bruzual and Spinrad (1978) and Kiang and Cheng (1982) realized that scanning magnitudes of QSOs across different redshift ranges led to an enormous discrepancy in estimates of q_0 , with dispersion of around 1.5. The result of the statistical analysis by Phillips and Ellis in 1983, the analysis which corrected for possible observational selection effects related to the choice of lines and instrumentation, was that the evolution of the cloud pressure was reasonable only for the smallest (less than 0.4) values of q_0 .

Once large amounts of data were available, the selection effect due to sampling of objects on photographic plates due to their brightness could be adequately assessed. The selection depends on the magnitude, and the spectral band it belongs to. The distribution of magnitude and the limiting values were taken into account in the analysis by Bigot et al. (1988). The expected Gaussian distribution of magnitude distribution in a comoving volume was tallied against the evolutionary values of luminosity functions via simulations, which then resulted in the estimate of $q_0 = -1.2$ based on the (corrected) data from Abell clusters.

As much as the diversity of instruments improves the measurements, the differences across instruments become more complex and have to be taken into account. A calibrating grid underlies the observational parameter grid, and finally the model grid with which the two are tallied.

Comparisons of different sources, namely galaxy clusters, detected with different instruments and observational programs posed a challenge early on (Sandage 1972). The more data were generated, the more acute was the need to standardize the observations and come up with the “world values” of basic parameters such as cluster ring counts, galactic density, determination of cluster centers, and so on. Moreover, the accuracy of photometric methods had to be estimated statistically over entire datasets, as well as internally through repeated measurements (across various conditions including seasons). The latter, however, differed substantially across these fairly rudimentary data samples (Sandage 1972, 11). At that point, standardization was inevitable and welcome, but small selection effects could inadvertently propagate due to such standardization. A standardization of counts of circles of galaxies within clusters had to be introduced rather than done on a case-by-case basis where systemic errors were very likely (Bruzual and Spinrad 1978).

There were two observational selection effects that plagued the cluster counts (Bruzual and Spinrad 1978). First, in distant clusters only the central part was visible, so the count depended on the use of the aperture (extra galaxies would appear with larger aperture) more than in the closer clusters. And a related effect was that the use of either of two methods for determining the size (direct or harmonic mean distance between galaxies) would lead to a substantial change when an extra galaxy was added. So the counts were really a function of aperture.

Although it was clear fairly early on (late 1960s, 1970s, or even in the 1920s with Baade’s work) that supernovas of a particular type can serve as standard candles due to essentially non-existent evolutionary corrections, their obvious rarity as well as the need for high resolution that could be achieved with space telescopes was a major obstacle for quickly obtaining reliable estimates (Wagoner 1977).

An even more specific assumption was explicated by Layzer (1962, 139) concerning the distribution of galaxies in clusters. It is the so-called virial theorem, stating that the distribution of galaxies in a spherically symmetrical cluster is itself spherically symmetrical. This can be averaged over a cluster or applied to a single galaxy. Similarly, the number of galaxies in a cluster ought to be proportional to the number density at the cosmological scale. So the count of galaxies and their radial velocities should represent the general cosmological expectation. Otherwise, q_0 could not be determined via galaxy counts and tallied with their radial velocities. It turned out, however, that the observational angle affects the measurements of the parameters substantially!

One purely observational dilemma in the late 1960s and early 1970s with substantial consequences concerned the spread of galaxies across redshifts (Noonan 1972). It was not clear whether to rely on the density per unit area versus the mean distance of the galaxies from the center of the cluster, that is, the galaxy ring count. The background density, statistical errors, and arbitrary choice of the cluster center plagued density techniques. The estimates of the linear radius of clusters varying with expansion within different relativistic models resulted in a range of q_0 from -1 to $+13$ (Noonan 1972, 136). As we have noticed, these ridiculously large positive values obtained in some cases could have been a contributing factor to the way of thinking which did not take physical cosmology seriously enough until at least the early 1990s. Obviously, observational selection effects were a major contributor.

4.5 Socio-epistemological

The inevitable institutional choices that, in turn, shape observational choices, and accordingly the kind of the results that will be produced, are affected by a combination of instrumental bottlenecks, observational time-allocation bias, general funding criteria prone to inertia (funding of more of the same), and citation inertia.

The timing of various telescopes seeing first light is certainly a major factor in the distribution and development of various observational techniques that we have listed and discussed. Initially, the measurements were tied to only a couple of telescopes, but the rapid growth of observational astronomy and the number of observatories suitable for cosmological observations led to the diversification of the techniques. The number of papers presenting results

based on different techniques indicates a pretty even distribution over time. Their number grew as the allocation times opened up, and as the techniques became viable. Yet this in itself is a bias of sorts that can hardly be avoided. Ideally, the most adequate techniques should have an advantage, and the even-handed spread across techniques may in fact slow the process down. Although it was clear that standard-candle measurements of supernovas were the most promising in terms of accuracy, it was hard to estimate to what extent sacrificing allocation time for galactic measurements would be beneficial, given the actual number of observable supernovas that also changed over time with the advances in resolving power. Whether tighter centralization or rather decentralization of allocation time across observational programs would have benefited the process is not an easy question to answer, given the maze of selection effects and assumptions inextricably linked to this socio-epistemological context.

On top of that, the funding inertia—funding more of the same, seemingly successful programs, at the expense of innovative and uncertain projects—has become an acute problem in astrophysics as it entered the world of Big Science. Also, given the sensitivity of the measurement outcomes to various results of other measurements, the citation inertia has been an unavoidable problem very early on. In fact, an early example where an old estimate simply was carried over uncritically to future research was pointed out by Sandage (1972, 8). A number of authors simply relied on the (m, z) number pairs provided in an early paper by Peach (1970) when he analyzed various other parameters. Depending on their specific analysis, the effect of this unchecked generic value affected their conclusions to various degrees. Moreover, Sandage's numerous papers themselves constitute a distinct long-lasting multi-decade self-supporting chain, and play an almost autonomous line of work in the network. Yet, they are the most cited papers in the network, which certainly raises the possibility of over-reliance on them in the community. This is in stark contrast to the papers of Tinsley's (and her coauthors) that raised early very-well-argued doubts on the positive value of the parameter but were not assimilated into the network for a long time the way Sandage's papers were.

We will integrate these aspects into our preliminary analysis at the end of our paper. In contrast to the controversy around H_0 , the controversy around the scatter of q_0 values listed in Table 1 was significantly more diverse in terms of both researchers and institutions involved. (To give just one example, many Chinese astronomers have consistently been engaged in doing observational work on q_0 , in contrast to the H_0 measurement history.⁴) But analyzing socio-epistemological biases in the historical context of the q_0 measurements merits an analysis of its own that we will undertake and publish separately in a much more developed form.

5 Tackling the assumptions

Along with the selection effects, there are cosmological and astrophysical general theoretical presuppositions, either explicitly advocated or lurking in the background mostly unnoticed, but either way directly affecting q_0 estimates. Cosmological models have relied on various general principles from the start. But not all theoretical presuppositions are as explicit and foundational as the cosmological principle. Some are more reticent, or even hard to identify or formulate but nevertheless very influential in swaying a pursuit at the observational and theoretical edge—the pursuit of q_0 —in a certain direction. Also, the importance of identifying various assumptions related to the basic astrophysical processes becomes crucial in a multi-method observational pursuit of a parameter like q_0 . The variety of techniques and lines of evidence in determining q_0 has made identifying these various assumptions ever more acute. Finally, these assumptions, as well as an essentially epistemological choice between numerous but complex and rare but fairly simple sources to analyze, are also extended into the complex institutional, socio-epistemological context of the search.

5.1 Cosmological

The solutions for the expanding universe depending on positive, zero or negative curvatures and the respective density of matter, correspond to the value of escape velocity (parabolic, less, or more than parabolic) constructed in the Newtonian framework. McCrea and Milne (McCrea and Milne 1934, 76) conceptualized the Doppler effect in the curved universe that should decrease with the observed epoch, as the universe decelerates with time due to the gravitational pull. They also note that in order for acceleration to take place, “a cosmic repulsion” (McCrea and Milne 1934, 79) that counters gravitational attraction must be at work. Now, the introduction of the *cosmological constant*, initially providing a static and closed universe, was understood as “somewhat artificial” even if relativistic expanding curved universes are constructed within the Newtonian framework. From the times of Eddington, it was clear that the cosmological constant has to be fine-tuned in order to provide for a static universe—and even

⁴ These contributions have been substantial all along, from the pioneering quasar-based measurements by the 1970s/80s USTC group, led by the renowned astrophysicist Fang Lizhi (see also Fang 2016), who later became one of China's best-known dissidents (Fang et al. 1976, 1982), to recent prominent results from the SLOAN collaboration and simulations on the Tianhe supercomputers led by Gong-Bo Zhao (Zhao et al. 2017).

after the universe was shown to be expanding, the association of λ with fine-tuning remained a stumbling block to its wider acceptance.

Hoyle and Sandage (1956), in their groundbreaking paper that argued for the realistic possibility of observing the change in the rate of expansion, H , over time by means of the redshift–magnitude method, stated that any possibility of acceleration would need to be supplemented by the cosmological constant of a particular value. Or more precisely, this move would take a form of accepting either a steady-state model (Bondi and Gold 1948) or the Lemaître–Eddington model with $\lambda > 0$ (Hoyle and Sandage 1956, 304). But Tomita and Hayashi (1963) recognized the potential importance of, at the time, “unfashionable” cosmological constant as a factor in prolonging the age of the universe (under the assumption of negligible evolutionary impact on apparent magnitude of galaxies), and also compared their theoretical conclusion with the existing redshift–magnitude values of galaxies.

In either approach, an accelerating universe was seen very early on as an option that requires a theoretically extravagant, and hence at least reticently less motivated addition of repulsion within the relativistic approach. Yet, already in 1970, it became clear that “as to the omission of the cosmological constant from the Einstein field equations, there is now some measure of agreement that such an omission is arbitrary” (Peach 1970, 753). And this judgment was drawn, at least in part, from the ever-growing results of photometric magnitude–redshift measurements and uncertainties surrounding them.

In 1975, Kollerstrom and McVittie tallied accumulated redshift–magnitude data analyzed by Sandage over the years to argue that they were compatible with the models with nonzero value of the cosmological constant. Given the incoming data that gradually hinted at the realistic possibility of negative q_0 , in the 1980s the inflationary theory was emerging, and examinations of various values of the cosmological constant in concert with fundamental physical assumptions followed. For instance, Klapdor and Grotz (1986) analyzed the relationship between the energy density of the universe and the cosmological constant, where negative values of q_0 emerged as a theoretically plausible solution.⁵

As far as cosmological assumptions affecting practical observations go, Hoyle and Sandage (1956, 306) offered a rough diagram of correlation of apparent magnitude and redshift based on four cosmological outcomes. In other words, they plotted what sort of change in the two values one ought to see based on each outcome. A similar but much more fine-grained diagram of the redshift–magnitude dependence with a range of only positive values of q_0 was published by Mattig in 1957. Finally, in 1962, Sandage (1962) gave numerical values of the apparent magnitudes and redshifts for various values of q_0 . In 1972, the data on redshift–magnitude were large enough (over 80 clusters) that it could be tallied with the models that vary in density. For three datasets of observed clusters, q_0 varied from -1 to $+3$ depending on the dispersion of redshift value (Sandage 1972, 20).

Layzer (1962) presented several key assumptions to consider in determining q_0 from galactic cluster counts and distances, some of which are relevant for some other or all methods. A key universally relevant assumption is the homogeneity and isotropy of the universe, in accord with Einstein–Friedmann models. Various violations of this assumption are treated as “special assumptions” (Layzer, 1962, 138) belonging to the models with a cosmological constant or to steady-state models. It is quite clear what was treated as an expected value of q_0 based on these assumptions. And the inflationary scalar field was practically placed not much further on the margin of the theoretical mainstream, as was the C-field of Hoyle in his steady-state account.

Yet, it was clear that whether and to what extent the universe may be “lumpy” (Sandage 1968) may affect the basic cosmological parameters. In 1969, Kantowski (1969) analyzed the assumption of a lumpy structure of the universe, ubiquitous local inhomogeneity modeled as “Swiss cheese,” in a (Einstein–Friedmann) model-neutral way and concluded that such lumpiness does not affect redshift or expansion velocity.

The correlation of matter density in the universe—varying across general relativistic models—and q_0 , was analyzed by Stabell and Refsdal (1966) across event horizons (including models with nonzero cosmological constant). The main goal of the paper was to establish the limits of the future horizon for the observations of q_0 . (Similarly, Sandage (1968) analyzed the distance between galaxies depending on the percentage of the elapsed evolution of the universe and q_0 , including a closed relapsing universe.) Haggerty and Wertz (1972) pointed out that due to anomalies in mass density both H_0 and q_0 may vary with time in a hierarchical universe. The local deviations in mass density would thus substantially affect the value of q_0 based on observations of magnitude–redshift correlations. And q_0 may be both scale-dependent and time-dependent as the homogeneity changes over time. They not only provided a theoretical background to the view but compared it with the existing redshift–magnitude observations, concluding that the negative value of q_0 should not be excluded. Also, the assumption that the universe was spherically homogeneous was questioned by Silk (1974). Large-scale spherical inhomogeneities may affect the Hubble expansion for an arbitrary value of q_0 . The local inhomogeneities of that sort could affect it even under pretty stringent conditions of shearing motions that produce it. The hypothesis was compared with the data on the Virgo supercluster of galaxies, with the conclusion that it is compatible with the data for q_0 of 0.5 or less.

McIntosh (1968) analyzed the impact of the assumptions on the ratio between radiation and matter content and pressure in the universe on q_0 . This is especially relevant to the early epochs as the dominance of radiation or matter varies substantially across the models. In 1986, Dabrowski and Stelmach (1986) analyzed redshift–magnitude

⁵ See also Ćirković & Samurović (2001) for a discussion of the cosmological constant and decaying dark matter model.

relations with respect to the radiation pressure, along with the previously examined value of the cosmological constant.

Since the discovery of the CMB, it became clear that its properties would coincide with particular values of q_0 . In fact, small values of q_0 (0 and below) would indirectly imply homogeneity of the universe via the presence of only small-scale anisotropies (Raine & Thomas 1981). The hypothetical large-scale anisotropies turned out to coincide with larger values of q_0 .

Possible relevance of dark matter to the value of q_0 as well as to its measurements was raised by Layzer in 1962 in relation to the brightness of galaxies and the overall known and unknown matter in the universe. He addressed it in the context of measuring the angular diameter of galaxy clusters, seeking theoretical parameters that are not affected by the ratio of matter density in bright galaxies and the universe in general.

Finally, perhaps the most outlandish questioning of the ubiquitous cosmological assumptions came from examination of the ratios of atomic radius and cosmological parameters (redshift, H_0 , q_0) following Dirac's large-numbers hypothesis. Chodos and Detweiler (1980) calculated the value of q_0 based on Dirac's ideas to be in the range between 0.73 and 1. Following this line of argument predicated on Dirac's hypothesis on the relation between the atomic radius and the radius of the curvature, as well as the relationship between the mass of the proton and the mass of the universe, but reformulated in Newtonian terms, Stoelinga (1985) suggested the value of 0 for q_0 . Not to be outdone, Raychaudhuri and Mukherjee (1984) looked at the so-called quadratic relation between redshift and distance which suggests that the universe is currently at a state of maximal expansion. If this is correct, the universe is closed under the density values that make it open under the standard assumption of a linear relationship between these two parameters.

5.2 Astrophysical

A very specific astrophysical assumption about galactic clusters pointed out by Layzer was that they are quasi-equilibrated and spherically symmetric to a sufficient degree (Layzer 1962, 138). Over time, this assumption has become substantially more complicated. And another seemingly innocent assumption about galaxy clusters was discussed and taken on board; namely, that at least the bright end of the galaxy clusters is the same as for galaxies in general, as well as that the ratio of density of matter in galaxies and matter in general is equally reflected both in clusters and galaxies. This is a typical example of how it was necessary to introduce assumptions at first that only subsequently looked quite coarse-grained.

Similarly, J. Richard Gott III and Gunn (1971) explored the mass and density of hot gas in the universe affecting q_0 in Friedmann models. This was a "hot" topic at the time, since the birth of X-ray astronomy a couple of years prior showed the existence of vast quantity of high-temperature ($T \sim 10^7$ K) plasma filling the entire volume of rich clusters of galaxies. Gott and Gunn put an upper limit estimate for q_0 at 0.1 based on X-ray measurements of the Coma cluster's gaseous properties, i.e., how much intergalactic gas could have entered the Coma cluster (i.e., infall of the intergalactic medium into the cluster). This was a rather preliminary estimate based on a limited dataset, but was theoretically well-motivated, explicated, and observationally informed nevertheless.

Eastmond (1965) analyzed the implications of stellar evolution for galactic evolution and q_0 . It turned out that the estimates of the evolution of stars can impact assumptions on the observable radiation across epochs, as radiation can be locked into unevolved stars like white dwarfs. Eastmond estimated that this sort of assumption makes any existing cosmological assessment based on observations substantially uncertain.

As far as evolutionary corrections go, in 1968 Sandage analyzed the assumption concerning the content of globular clusters populated by elliptical galaxies alone. The mass-to-light ratio did not agree (an early hint of dark matter, along the prescient ideas of Fritz Zwicky), nor did the metallic content of stars in elliptical galaxies tally with the metallic content in globular clusters based on spectral analysis. It was clear at that point that figuring out evolutionary corrections would not be as straightforward as desired.

Populations of galaxies within clusters possibly varied in time due to evolutionary processes, so the counts relying on selective redshifts could be substantially unreliable (Noonan 1971, 137). Also, it was clear that the observed correlation of size, concentration, and population of clusters across epochs had to be compared across cosmological models (Austin and Peach 1974).

The results of the galactic counts, their redshifts and colors, and the extragalactic background across all galaxy types were compared across various galaxy types and tallied via computer simulation with various q_0 values by Yoshii and Takahara (1989). It was obvious that the redshift-magnitude method suggested high q_0 values, as opposed to lower values resulting from the galaxy counts. Now, the two key assumptions used in tallying these results with the q_0 were, first, brightening early in the evolution of galaxies, and second, the epoch of galaxy formation (at redshift 3–5, quite late by today's standards). The estimated value of q_0 was 0.5.

Substantial intergalactic dust, modeled in a Friedmann universe homogeneously filled with it, would affect measurements of q_0 substantially. A 0.30 magnitude change would affect q_0 by about 0.5 (Meinel 1981). The modeling demonstrated that intergalactic dust could be a significant factor in the final tally of the parameter. Of

course, such dust would have profound effects on the total baryonic tally of the universe; newer research has shown that the amount of dust in the intergalactic space is indeed negligible.

5.3 The statistics versus sensitivity trade-off

An inevitable and central assumption in estimating q_0 observationally stemmed from the key difference in the number of sources used for the redshift–magnitude method on the one hand, and the standard-candle and standard-ruler methods on the other. The former relied on a large number of galaxies and clusters of galaxies, accessible by state-of-the-art telescopes used for measurements, but was vulnerable to evolutionary corrections. The latter, the use of supernovas (and initially first-ranked cluster galaxies and QSOs) as standard candles, and SZ sources as standard rulers, eliminated the need for extensive evolutionary corrections but suffered from haphazard accessibility of the sources and—in the case of QSOs—from their poorly understood physical properties. Thus, the sensitivity of observations to evolutionary and other assumptions based on the plethora of data was pitted against the low numbers of standard candles (and rulers) measurements. The variations in values of key parameters used for the inference of q_0 in the case of the former were much wider than in the latter case, which only emphasized rather than resolved the dilemma. Unsurprisingly, an adequate level of reliability was achieved only after a long period of accumulation of a sufficient number of relevant supernova measurements.

6 Why q_0 is historically and theoretically more interesting than H_0

Observational discrepancies and theoretical disagreements over the value of H_0 were in effect a simple polarization between the two schools of thought (and, perhaps, relatively slight differences in understanding the best way of doing cosmology); however, these controversies are rather tame in comparison with those in the effort to determine the value of q_0 and understand its exact theoretical meaning.

Also, the consequences of a change in the numerical value of H_0 have for quite a long time been understood as cosmologically mundane—even a substantial difference in value does not really change the basic cosmological premises of the Big Bang models embedded in the general relativistic field equations. Yet, whether the universe decelerates, is stagnant, or accelerates concerns the most basic assumptions of cosmological models. In a sense, the value of the parameter makes sense of the most controversial, and yet most appealing subject matter, the *future of the universe*. The expectation that the value of q_0 would inevitably turn out to be positive was a result of the wide acceptance of the basic assumptions of the hot Big Bang model and, arguably, mundane equations of state (giving pressures $P \geq 0$ for all kinds of cosmological matter). On top of this, multiple observational discrepancies across various lines of evidence made things even more complicated, as they produced consequences across cosmological and astrophysical parameters, leading to complex debates over these basic cosmological assumptions. In fact, wild discrepancies in measuring q_0 may have been a major contributing factor to the widespread skepticism toward physical cosmology as a legitimate scientific field. It certainly did not help that a field-defining parameter varied from -1 to 13 or more across measurements, especially when considering the widely different implications of this in terms of the fate of the universe the field was trying to get a grip on.

So, there are both intrinsic (theoretical) and historically contingent differences between the H_0 and q_0 controversies; these have not been studied in the literature so far. While the historically contingent differences have been studied throughout this paper, here we just briefly review the main intrinsic differences. Three main items making the deceleration parameter intrinsically more appealing for a historical and methodological review than H_0 are elaborated below.

First, as the second-order term in the expansion series for the scaling factor (or redshift), q_0 is less connected to run-of-the mill astrophysics than H_0 . In particular, the fact that the critical cosmological density at present Ω_0 features the Hubble constant means that any cosmological measure of energy will indirectly include H_0 . This can be seen in its explicit appearance in the Friedmann equation; consequently, it branches into a plethora of relevant astrophysical formulae. In contrast, the deceleration parameter features in a significantly smaller number of numerical relationships. Of course, that fact is a part of the explanation why more historical and philosophical attention has been devoted to H_0 than to q_0 so far, the state of affairs we modestly attempt to amend in the present study. By the very same token, however, the domain of applicability of q_0 is more severely circumscribed and there are fewer extraneous factors necessary to take into account. This makes studying q_0 in the historical context much more self-contained and well-defined than studying H_0 . This is also why the q_0 controversy has reached its well-defined endpoint, something that is in fact required for a proper historical account (see the detailed and beautiful discussion in Kragh 1996), which cannot be said for H_0 .

Second, q_0 is a dimensionless real number of the order of magnitude unity. This formal property is actually quite important in light of the theoretical desiderata for physical theories (e.g., Duhem 1906; Godfrey-Smith 2003; Maudlin 2007). H_0 , while having formally the dimensions of $(\text{time})^{-1}$, has usually been expressed in cumbersome

units of $\text{km s}^{-1} \text{Mpc}^{-1}$, that is, velocity/distance-in-megaparsecs, obviously convenient for extragalactic observational astronomy, but not very intuitive, and arguably somewhat confusing in regard to the underlying physics. This advantage of q_0 has been implicitly acknowledged by accepting the conventional parametrization of Hubble's constant as:

$$H_0 = 100h \text{ km s}^{-1} \text{Mpc}^{-1}, \quad (1)$$

where the “small h ” is now a dimensionless quantity of the order of unity, on a par with q_0 (although the subscript “0” has been dropped for the small h , it is still a function of cosmic time in most cosmological models and it is understood that the h is evaluated at the present epoch). Confusingly enough, a reference unit other than $100 \text{ km s}^{-1} \text{Mpc}^{-1}$ may be chosen; in particular, a unit of $70 \text{ km s}^{-1} \text{Mpc}^{-1}$, in which case h with a subscript is deployed, e.g., h_{70} . (The controversy about the value of Hubble's constant in the real universe did not cease with the parametrization, of course; as mentioned above, it is now usually referred to under the fancy name of the “Hubble tension,” cf. Freedman (2021); Kamionkowski & Riess 2023). Fiducially, we use the value throughout the present paper (Aghanim et al. 2020; Balkenhol et al. 2023). The special position of the expansion rate is reflected in the dependence, often quite sensitive, of all cosmological values on the “small h ,” i.e., the Hubble parameter.

Finally, q_0 has a straightforward, intuitively clear interpretation, in contrast to H_0 : It means the expansion of the universe is slowing down due to the gravitational attraction of normal matter ($q_0 > 0$), or the expansion is accelerating due to the presence of some exotic form of matter ($q_0 < 0$), or it corresponds to the expansion coasting, i.e., occurring adiabatically without significant gravitational impact either way ($q_0 \approx 0$). As it turns out, in the absence of dark energy (or any other exotic matter), positive q_0 is linearly proportional to the density of non-relativistic gravitating mass, with a boundary case between eternally expanding and eventually re-collapsing Friedmann models. Bigger q_0 means more gravitating matter in the universe; a value of q_0 close to zero means nearly free, adiabatic expansion of the universe; the more negative q_0 is, the more exotic matter there is, causing the accelerated expansion (see Appendix 2). This is all plain and simple—it requires an entirely different level of astronomical training and insight to discern the implications of H_0 being $55 \text{ km s}^{-1} \text{Mpc}^{-1}$, as opposed to, say, $95 \text{ km s}^{-1} \text{Mpc}^{-1}$. (Smaller discrepancies characterizing the last three decades, roughly since the launching of HST, are even more difficult to correctly interpret, requiring invocation of inhomogeneities, large-scale streaming motions, the influence of the local supercluster and similar complex cosmological paraphernalia.)

These circumstances reveal another important motivation for studying q_0 : our very much alive quest to understand dark energy or what is occasionally and somewhat pompously called “the dark energy sector” of cosmology (e.g., Turner 2018; Di Valentino 2021; Menci et al. 2024).

Apart from the dearth of historical analyses, there is another reason to contrast the fire and fury, as well as the theater surrounding the determination of H_0 with the comparative silence about its twin key number, q_0 . In the history of H_0 controversies, there are clearly defined phases and trends. The first was between Hubble's discovery of the expansion of the universe and Baade's revision of the distance scale (1929–1952) when an unrealistically high value for H_0 persisted and caused all kinds of conflicts with the ages of various celestial bodies and with geophysical evidence. Subsequently, there have been two largely self-consistent schools of thought—of “low H_0 ” (Sandage, Tammann, et al.) and “high H_0 ” (de Vaucouleurs, van den Bergh, Freedman, et al.). In spite of the H_0 debate being often quite fierce and uncompromising—especially in the 1980s and early 1990s on the eve of the *HST* launch and its first dramatic repair in 1993—it has always been a highly structured affair. In terms of many modern reviews (a good example is Kamionkowski & Riess 2023), the history of H_0 is the history of multiple “Hubble tensions” between apparently incompatible observational results and, occasionally, theory-laden astrophysical facts like the age of the oldest stars (which offers the logical minimum for the age of the universe as a whole). The newest bout of the same tension unfolds between well-defined categories of the “early universe” and “local” H_0 measurements.

There is no such systemic consistency in measurements of q_0 . Not only can no clear diagram of the historical evolution of q_0 be drawn (at least without it looking like white noise), but consecutive measurements, often by the very same techniques, or even the same researchers in the span of a few years have often yielded very different results, sometimes incompatible on both 1σ and 3σ levels. This has had another important consequence for the history and methodology of cosmology: In contrast to the H_0 case and similar issues like the CMB interpretations (Perović & Ćirković 2024; Ćirković & Perović 2018), in this particular case of q_0 there has been no “school” or “tribal” loyalty to a particular group or school or style of research. This situation cracks wide-open an assortment of selection effects and assumptions, as well as their mutual amplifications.

Our analysis is a historical, methodological, and philosophical study of the pursuit of one of the key cosmological parameters at the very limit of observability and at the edge of theoretical insight. It is part of an effort to understand common features of such cutting-edge pursuits in cosmology across key cosmological parameters.⁶ The study aims to pave the way for the applicable lessons for the ongoing and future explorations of other constants

⁶ See an early shot at the topic in Davidson (1962).

and key parameters in cosmology. Our future goal is to undertake an explicit comparison with the dark energy parameter by building a general grid and selection effects/assumptions indicators based on the case of q_0 .⁷

7 Everything evolves!

The main reason for extreme—or scandalous—scatter of empirical results on the measurement of q_0 is usually glossed over as “evolutionary uncertainties.” This is one of the terms so often used in passing that its actual meaning has gradually become quite mysterious. So, what do “evolutionary uncertainties” really mean?

In observational cosmology, it is both beneficial and detrimental that we look back into the distant past. While benefits are often loudly proclaimed, the negative aspects are pushed under the rug, which sometimes causes problems down the line. A major problem is that the sources belonging to the same category (galaxies or clusters or QSOs or absorption line systems) are observed at wildly different ages, hence in wildly different stages of their intrinsic physical, chemical, and morphological evolution. This makes their comparison—and their inclusion into a statistically significant sample—quite difficult and reliant on some theoretical assumptions which cannot be either derived “from first principles” or independently confirmed by a different set of observations. This is especially pronounced in those cases in which the physical nature of a source is unclear or conjectural—which was, for instance, the case with QSOs for quite a long time (and quite a long chunk of the 1952–2010 time window studied here), until it was finally confirmed that they are bona fide active galactic nuclei (e.g., Stockton 1978; Gehren et al. 1984).

We have seen how the detrimental combination of evolutionary selection effects and assumptions and observational selection effects led to reliance on measurements of q_0 based on galaxies. Yet this should not come as a surprise as even normal galaxies are quite problematic in the context of observational cosmology, for multiple reasons. Probably the most important source of epistemic difficulties is their sheer physical size and complexity. This may sound strange in light of the fact that even in the best and deepest galaxy surveys, distant galaxies (say those at redshifts $z > 0.1$, corresponding to a comoving distance larger than about 420 Mpc in the concordance cosmology) are just faint little smudges on the CCD camera or on a screen. Still more distant galaxies may look, especially to an untrained eye, entirely star-like in their visual appearance. Yet it is deeply misleading, since we know that those are Kant’s and Hubble’s “island universes,” huge systems comprised of between a few million and more than a trillion stars, interstellar matter in its various and complex phases, innumerable smaller objects like planets, moons, asteroids, comets, and dust grains, as well as a huge amount of non-baryonic dark matter, the details of whose distribution are still unclear. In addition to this compositional complexity, there is a bewildering *historical* complexity supposedly practically reducible to evolutionary correction, since in contrast to simpler systems like individual stars or gas clouds, history matters for galaxies, especially in view of their fluctuating environment and multiple mergers with other galaxies or their close passes, strongly influencing their observable properties. Essentially, the “island universe” label is proving to be much more than a metaphor: It is an indicator of extreme complexity and contingent, non-ergodic evolution of these huge systems. Even at our nearest neighborhood, we encounter difficulties stemming from the same source: Our own galaxy, the Milky Way, and M31 seem very similar at first glance, both being large spiral galaxies, but in fact are quite different in evolutionary terms, being on the opposite sides of the “green valley.” (For instance, the net star-formation rate in M31 is 3–5 times smaller than in the Milky Way in spite of M31 being 2–3 times *larger* in terms of stellar mass and spatial extent.)

Altogether, this makes treating galaxies as “standardized” objects or sources a fool’s errand (Dick 2015). This does not apply just to q_0 measurements from various samples of galaxies. Other cosmological projects faced the same obstacle: A famous example is the “Seven Samurai” attempt to measure the peculiar velocity of the Local Group which resulted in the discovery of the Great Attractor (Dressler 1987; Dressler et al. 1987). A large part of the uncertainties consisted in the *inhomogeneity* of the sample of elliptical galaxies sufficiently symmetrically distributed around the Milky Way. A deeper theoretical problem in determining the best option was and still is the fact that the needed precision of measurements may be adversely affected by the inherent diversity of galaxies. The H-R-style diagram that classifies galaxies and their evolutionary rates may be too rough a generalization for such a purpose (Salim 2014). And perhaps galaxies can be properly treated as taxa. They are after all, when observed at a deeper level, too big and too different to be simplistically classified, unless we need a rough and ready classification. That circumstance may have proved to be an insurmountable obstacle for q_0 measurements based on galaxies.

⁷ It would also be advantageous to compare the general epistemological context with the high-energy-physics context, as a heavily experiment-based field, in order to situate the cosmological pursuit, the power and limits of its methods and theoretical insights, within the fields of physical sciences more broadly. This opens up less obvious dilemmas for experimental sciences concerning reproducibility.

8 Confronting the maze: assumptions, selection effects, and epistemic trade-offs

Yet the assumptions plaguing the attempts to nail down the evolutionary correction to the redshift–magnitude diagram were only one set, albeit an important one, among many. Generally speaking, we rarely see a glimpse of a polarizing pattern in the literature. Rather, the assumptions, just like selection effects, had been diffused all along. Moreover, the interaction between such strong theoretical assumptions, epistemological and social–epistemological trade-offs, and the diversity of selection effects was responsible for a very diffused disagreement of results. On the one hand, selection effects over time reflected the development of various detecting techniques. On the other, the development of theoretical views, especially those relevant for evolutionary selection effects, was equally relevant.

Linder (1988) nicely summarized and analyzed the issue of selection effects that became apparent over time. He offered three heuristic methodological rules for avoiding astrophysical selection effects across observational techniques for determining q_0 via redshifts:

1. *Simplicity*: avoiding sources with strong evolutionary effects.
2. *Accuracy*: pursuing observations with low dispersion, easy to grasp theoretically.
3. *Clarity*: easy understanding of potential errors that may accumulate.

Rule number 1 concerns the nature of galaxies and galaxy clusters which are readily available and gestures to the use of a standard-candle strategy. Rule number 2 suggests the plethora of theoretical biases and assumptions we have identified that ought to be avoided. And rule number 3 points out various sorts of inertias leading to carrying over assumptions and selection effects from one measurement to another. This list, formulated fairly late in the game, is a tall order to follow. The historical trajectory shows how the general strategy of rendering techniques in various directions was followed instead of narrowing them down in accordance with these rules. Even though following the latter strategy may have been a winner for now—with the use of the supernova standard-candle techniques checking all three boxes—this focus could not have been possible without widespread rendering of techniques that led to such a choice and its results in the end. It is also clear that this is not an endpoint but only a significant point in multiple trajectories along which modern observational and theoretical cosmology has been developing.

As far as the domain of assumptions and biases goes, in fact, a range of values of the parameter could be based on the same data by making different assumptions at pretty much any given point from the early 1970s to the 1990s. And that had become quite obvious in the mid-1970s as Gunn and Tinsley (1975) and Chitre and Narlikar (1976) demonstrated.⁸ There were two kinds of theoretical assumptions; those argued and those assumed and implicit. And on top of that, the socio-epistemological patterns are either rooted in the theoretical assumptions, or the result of rather theory-neutral inertia. It is not easy to confirm that latter is the case when explicit theoretical commitments are not voiced. In the diffusion of theoretical assumptions, biases, and the overall theoretical maze, the cosmological constant stands out as a kind of a lever for formulating explicit hypotheses built in the models or observational programs very early on, despite a generally unfavorable view of its cosmological relevance. It should also be pointed out that particular cosmological assumptions affect the observational programs and measurements, and the way they were explicated and mapped out very early on, especially starting in the early 1950s with subsequent refinements reflecting developing techniques and fine-grained distinctions they enabled.

Finally, it should have become transparent by now that opting for one of the two obvious trade-offs drove the discrepancy in the measurement results. First, the trade-off between readily available galaxies for the redshift–magnitude technique and the evolutionary correction, and second, between the supernovas and Sunyaev–Zeldovich (SZ) sources as standard candles and their meager numbers. Yet, less visible but also an overarching factor in opting for one side in these trade-offs was the speed of technical developments leading to new techniques and trade-offs, which also requires a deeper analysis of its own, as an inevitable aspect of any Big Science pursuit of basic cosmological parameters. Comprehensively analyzing the socio-epistemological biases in the history of q_0 measurements, and all the implications of their inextricable link with the basic cosmological and astrophysical assumptions, warrants a more developed study—hopefully applicable to other cosmological measurements (e.g., dark energy)—which we will publish separately.

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Author contributions Both authors whose names appear on the submission made substantial contributions to the conception and design of the work, the acquisition, analysis, and interpretation of data. They drafted the work or revised

⁸ See also more recent De Leon 2006.

it critically for important intellectual content, approved the version to be published, and agree to be accountable for all aspects of the work in ensuring that questions related to the accuracy or integrity of any part of the work are appropriately investigated and resolved.

Declarations

Conflict of interests The authors declare no competing interests.

Appendix 1: q_0 and other cosmological parameters

As we have mentioned earlier, Sandage (1970) summarized the interrelation between the predicted parameters in a poignant way by stating that the entire cosmology is in search of two numbers: the Hubble constant telling us how fast the universe is expanding, and q_0 , telling us how fast it decelerates under the gravitational pullback. Within the Friedmann–Robertson–Walker metric, the most general global metric compatible with large-scale homogeneity and isotropy of the universe, the line element in spherical coordinates (r, θ, φ) is given as (Peacock 1999; Islam 2004):

$$ds^2 = c^2 dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right], \quad (2)$$

where $R(t)$ is the scale factor.⁹ Roughly speaking, it describes how the distance between two randomly chosen observers changes with time. Both H_0 and q_0 follow from the Taylor expansion of the scale factor around present time t_0 :

$$\begin{aligned} R(t) &= R[t_0 - (t_0 - t)] = R(t_0) - (t_0 - t)\dot{R}(t_0) + \frac{1}{2}(t_0 - t)^2\ddot{R}(t_0) - \dots = \\ &= R(t_0) \left[1 - (t_0 - t)H_0 - \frac{1}{2}(t_0 - t)^2 H_0^2 q_0 + \dots \right]. \end{aligned} \quad (3)$$

That is, the two key observables are defined on the basis of first and second derivatives of the scale factor evaluated at the present epoch:

$$H_0 = \left. \frac{\dot{R}(t)}{R(t)} \right|_{t=t_0}, \quad (4)$$

And

$$q_0 = \left. \frac{\ddot{R}(t)R(t)}{[\dot{R}(t)]^2} \right|_{t=t_0}. \quad (5)$$

We immediately perceive the origin of the approximate linear relationship between redshift and distance, known as the Hubble law: It is simply a consequence of taking into account only the term linear in $(t_0 - t)$ in the expansion (3). The *deviation* from the linear law on the redshift-distance plot is due to the higher-order terms—notably the term containing q_0 .

The most important *derived* equation for q_0 is:

$$q_0 = \frac{\Omega_m}{2} - \Omega_\Lambda, \quad (6)$$

⁹ In some cosmology textbooks, a distinction is made between the scale factor $R(t)$ figuring in the FRW metric and the normalized scale factor $a(t)$ expressed in units of the present-day scale factor, that is $a(t) \equiv R(t)/R(t_0)$. This has an obvious advantage in that the present-day scale factor is set to unity and hence vanishes from some more complicated cosmological calculations (e.g., those pertaining to the power spectrum and growth of gravitational perturbations). Since the normalization factor is completely irrelevant for our present purposes—being the same for all derivatives in the listed equations, hence canceling out—we shall retain $R(t)$ throughout.

where Ω_m and Ω_λ are dimensionless densities of matter and dark energy, respectively, defined by:

$$\Omega_m = \frac{\rho_m}{\rho_{crit}} = \frac{8\pi G}{3H_0^2} \rho_m; \quad \Omega_\lambda = \frac{c^2}{3H_0^2} \lambda. \quad (7)$$

Equation (6), very relevant for the observational results discussed in this paper—and much of the confusion resulting from observational errors—is derived from the Friedmann equation:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho + 3p), \quad (8)$$

where p is the pressure exerted by matter with total density ρ , both normal and exotic. It is the insight that the exotic matter in any form (dark energy, Einstein's cosmological constant, quintessence, phantom energy, topological defects, etc.) exerts *negative* pressure which will lead to an accelerated expansion of the universe at later epochs that gives rise to Eq. (6).

Thus, the predictions of cosmological models, in particular the Hot Big Bang (HBB) model, were focused on the curvature of space, time since the creation of the universe, and the averaged content (matter and radiation) of the universe. Similarly, the so-called Hubble expansion rate (measuring the time since creation) is dependent on the density (the averaged content) and the curvature. The observational parameters that test the predictions of these basic theoretical parameters have steadily multiplied since Hubble's measurements of the spectral redshift of galaxies due to their recession. And as far as the details of the actual tests of key observational parameters go, they were “kept as a black art” (Sandage 1988) in the words of Allan Sandage, one of very few key cosmologists working on q_0 measurements over several decades.

Appendix 2: Summary of cosmological models

Table 2 Scaling factors and the values of q_0 as predictions of some simple cosmological models of historical importance. Only the most typical references describing each model are provided here, irrespective of whether they specifically discuss the deceleration parameter. Some models have additional parameters, such as either equation-of-state parameters of the postulated exotic matter or the transition redshifts in #10 and #11, on which the calculated values of q_0 (as per Eq. (5) above) depend

No	Cosmological model	Scale factor $R(t)$	Prediction for q_0	Reference	Comments
1	Einstein-de Sitter ($\Omega = \Omega_m = 1$)	$\propto t^{2/3}$	0.5	Einstein & de Sitter (1932)	Preferred theoretical model in most studies in the 1965–1998 period
2	Einstein static universe	const	none	Einstein (1917)	Since there is no expansion, it makes no sense to talk about the deceleration rate! ^a
3	de Sitter	$\propto e^{Ht}$	− 1	de Sitter (1917)	Empty universe with cosmological constant
4	matter-dominated low density ($\Omega = \Omega_m = 0.2$)	$\propto t^{2/3}$	0.1	Coles & Ellis (1994)	What has actually been seen in the 1965–1998 period: too little gravitating mass to appreciably slow down the expansion
5	radiation-dominated flat ($\Omega = \Omega_\gamma = 1$)	$\propto t^{1/2}$	1	Kolb & Turner (1990)	Generic model for the very early universe ($z > 3600$)

Table 2 (continued)

No	Cosmological model	Scale factor $R(t)$	Prediction for q_0	Reference	Comments
6	Eddington–Lemaître universe	$\text{const.} \rightarrow e^{Ht}$	$0 > q_0 \geq -1$	Lemaître (1931)	Combination of the Einstein static and de Sitter’s exponentially expanding model. The exact value of q_0 will depend on the magnitude of cosmological constant and how far are we from the start of the expansion
7	Milne universe	$\propto t$	0	Milne (1935)	A special-relativistic FRW universe with negative curvature; also known as “kinematical relativity”
8	classical steady-state	$\propto e^{Ht}$	-1	Hoyle (1948); Bondi and Gold (1948)	Formally the same metric as the de Sitter model, but with continuous creation of matter. The main rival to the standard Big Bang cosmology in the 1948–1965 period (Kragh 1996)
9	Dirac’s varying G	$\propto t^{1/3}$	2	Dirac (1974); Davidson (1982)	Similar to the “normal” Big Bang models, except for the varying G effects, mostly manifested locally. Rapid deceleration is due to the increase of the mass contained within cosmological horizon: $M \propto t^2$
10	coasting universe	$\propto t$	~ 0	Kolb (1989); Akdeniz et al. (1994)	Curvature term dominates, in contrast to all other models listed; the <i>exact</i> value of q_0 depends on the redshift of the transition into the coasting regime
11	Lemaître’s “loitering” universe	$\text{const.} \rightarrow e^{Ht}$	-1.3	Kardashev (1967); Sahni, Feldman & Stebbins (1992); Elgarøy et al. (2005)	Expanding universe with an Eddington–Lemaître “loitering” phase before transition into exponential expansion. The value of q_0 depends on both the equation of state of the exotic matter and the redshift of the loitering epoch

Table 2 (continued)

No	Cosmological model	Scale factor $R(t)$	Prediction for q_0	Reference	Comments
12	New Standard (Λ CDM) model	$\rightarrow e^{Ht}$	-0.6 ± 0.2	Ostriker & Steinhardt (1995); Camarena & Marra (2020)	Dominant since 1998. The uncertainty listed applies to purely empirical data. With the assumption of global flatness $\Omega = 1$, this uncertainty is significantly reduced

^aThis applies, of course, to all other static models, like the tired-light models (e.g., Sorrell 2008)

Appendix 3: Observational methods

In this Appendix, we will describe in very general and fairly non-technical terms, going only into the most necessary key technical details, each of the techniques used in measurements of q_0 over several decades. We will turn to the sticking points and selection effects for each of them in subsequent sections.

The magnitude and redshift of galaxies

The relationship between absolute luminosity (the actual brightness of the object) and the redshift has been used the longest as a parameter for determining q_0 . Going back to Hubble and deployed by his students Humason, Mayall, and Sandage for this purpose in 1956, it is the most readily available method due to the abundance of the sources (mostly galaxies), but it is also an inherently complex relationship. It essentially aims at measuring the increase/decrease of velocity (via redshift) of spatially sequential objects whose distance is determined by their luminosity, and comparing differences with those predicted by cosmological models (Table 2). Obviously, the precondition to the application of this method is that the *extragalactic distance scale* is properly calibrated: the precondition which was not satisfied until Walter Baade corrected Hubble's original mistake in confusing two different kinds of Cepheid variables (1952). The determination of the luminosity of galaxies, however, is closely related to their evolution: luminosity of normal galaxies in the optical comes from their stellar population, which evolves over time.¹⁰ Thus, the apparent magnitude has to be appropriately adjusted to the evolutionary stage of the object. As elliptical galaxies have the most stable evolutionary curve, they have been used as standard objects for such measurements; the difficulty here is that elliptical galaxies are mostly either small and faint (hence impossible to detect at cosmological distances) or located in dense galaxy clusters (where their evolution is strongly impacted by interaction with their environment, although this has not been much appreciated until the advent of modern numerical simulations in 1990s).¹¹

The same method is applied to the measurements of quasi-stellar objects (QSOs) as well as to supernovae. Type Ia supernovae have an almost constant absolute magnitude (essentially depending on the value of the Chandrasekhar mass, which only moderately varies with metallicity of the exploding white dwarf), implying the advantage of using them as standard candles: All variation in their apparent magnitude comes from the distance (hence, expansion of spacetime occurring during the transit time of the light signal). This luxury is not available in galaxy measurements due to their complex evolution. On the other hand, galaxies are very numerous, while supernovae, especially at large distance, are rare and unpredictable.

The galaxy counts and magnitude

Counting galaxies in a series of limiting magnitudes is compared with the predictions of the models of galactic magnitude changes across epochs within given volumes (Ott 2005). The value at stake is basically the same as in the redshift–magnitude method, namely the luminosity–distance function within a volume of space. The underlying assumption there is the number of galaxies per unit comoving volume is conserved, at least for sufficiently late epochs (when the process of galaxy formation is presumably over). The key challenge is, similarly, to adequately

¹⁰ Central nuclear engine contributes very little if anything to the optical luminosity of normal galaxies (active galaxies are a small minority which is easy to spot and exclude from the survey, at least at low redshifts—which is all that was available to observational astronomers up to early 1990s).

¹¹ An additional difficulty is that many famous ellipticals like M87 or Centaurus A (NGC 5128) manifest significant nuclear activity.

tally the observations with the evolutionary correction, which here comes from the phenomenon of galaxy mergers, also appreciated sufficiently after the advent of large-scale N-body cosmological simulations (cf. the seminal study of Lacey & Cole 1993).

The color and redshift of galaxies

A more fine-grained parameter used in combination with redshift data is the analysis of colors—i.e., analysis of a spectral line or a band of spectral lines in the spectrum of a galaxy in order to determine the object's evolutionary stage, and thus make an accurate estimate of the absolute magnitude and distance. Along with the redshift measurements, this is then used to compare the velocities of galaxies at different (close) distances in order to determine q_0 .

Supernovae as standard candles

There is a long history of using supernovae as standard candles, in order to determine the distance to galaxies. The brightness of a supernova far exceeds that of the galaxy in which it is located. The very quick evolutionary trajectory of a supernova is far simpler and more uniform than that of any galaxy, and with some luck, we can actually see the entire supernova's short evolutionary trajectory. This means that estimates of the distances of supernovas are essentially analogous to estimating a distance from a stable (non-evolving) standard candle by its apparent magnitude as it is moving away from us. Knowing their distances, the differences in redshifts of supernovas (and their respective galaxies) then give us the q_0 value.

Galaxy counts in clusters (number and magnitude)

Different values of q_0 lead to different expectations for the abundance of galaxies at given epochs (redshifts), which can be compared with observations. Thus, within a given volume, an expected temporal distribution of galaxies can be tested by counting the number of galaxies at each redshift value. Thus, simple counts of galaxies on photographic plates, within various clusters at various (preferably close) redshifts, ought to test the predictions for q_0 .

X-ray sources counts

Differential number counts of cosmologically distributed X-ray sources were proposed along the lines of the galaxy counting method. The method is essential the same, with a difference of substantially less sources being available, as well as their physical nature being much less clear in general case.

Angular size of clusters and distribution of galaxies

In general, due to the expansion of the universe, objects such as galaxy clusters should be ever-larger. Their apparent angular size should decrease with increasing redshift. This also implies that the further in the past we look across redshift values the closer the galaxies should be to each other within galaxy clusters. Yet, this is true up to a point, as the density at some point decouples from the Hubble flow, when objects become gravitationally bound ("turnaround"); subsequently, their size tends to *decrease* in the local frame of reference. So this simple expectation requires turnaround point corrections.

The CMB (Sunyaev–Zeldovich effect) interaction with intergalactic and galactic clouds

The Sunyaev–Zeldovich effect (SZ) due to the cosmic microwave background radiation (CMB) attenuation enables various objects to serve as standard rulers (rather than as standard candles). The omnipresent remnant of radiation, now at 3 K peaking in the microwave region of the spectrum, created by the primordial Big Bang fireball, interacts with various objects, such as intergalactic gas clouds. These interactions result in the SZ effect, whereby, due to the inverse Compton scattering, the energy released from the interaction is basically transferred from the object to the CMB, where the number of photons oscillating at higher frequencies increases at the expense of a detectable loss of those oscillating at regular lower frequencies—the attenuation of the CMB at such lower frequencies compared to the usual measurements, at the line of sight of the SZ objects, is detected. Crucially, the outgoing attenuated radiation is not a result of thermal processes that reflect the evolutionary stage of the object, but of the interaction with the CMB (i.e., the CMB's radiation attenuation at the line of sight of—i.e., "coincidental with"—the object). Thus, the change in the magnitude of the same sort of objects (X-ray sources) across redshifts can serve as

a standard ruler. Finally, the velocity change measured by redshift (X-ray measurements) can be standardly compared with the magnitude change in order to determine q_0 .

This effect has been exploited across various sorts of objects, but initially, the idea was to look at the extra-galactic background across the skies using the SZ effect as a way of bypassing evolutionary effects in a way. Intergalactic clouds that emit radiation mainly in the X-ray region of the spectrum are a suitable target for SZ effect measurements. Also, the hot gas cores of clusters of galaxies have been proposed and used in the same vein.

QSOs

Out of all the methods that attempted to use particular objects as standard candles, the attempts to use quasi-stellar objects (QSOs) have been the most controversial. The discovery of their redshift distribution, greatly peaking at around $z = 2$, was treated as a cosmological property; QSOs are objects generated mostly at a certain epoch, and their numbers abruptly decrease in time. Thus, their evolution should be much simpler than that of galaxies, which are long-lasting, generated for much longer periods of time, and vary greatly in structure across time. The QSOs' apparent magnitude then should be close to their absolute magnitude (reflecting their true luminosity). The initial hope was that their redshifts straightforwardly measure their velocity. Yet, the interference of their evolution as active galactic nuclei with the redshift measurements, has been a subject of controversy in the decades we analyze.

References

- Aghanim, N., Akrami, Y., Ashdown, M., Aumont, J., Baccigalupi, C., Ballardini, M., Banday, A. J., Barreiro, R. B., Bartolo, N., Basak, S. & Battye, R. (2020). Planck 2018 results-VI. Cosmological parameters. *Astronomy & Astrophysics* **641**, A6 (67pp).
- Akdeniz, K. G., Arik, M. & Rizaoglu, E. (1994). Coasting big bang. *Physics Letters B* **321**, 329-332.
- Astier, P., et al. (2006). The Supernova Legacy Survey: measurement of Ω_M , Ω_Λ and w from the first year data set. *Astronomy & Astrophysics* **447**, 31-48.
- Austin, T. B. & Peach, J. V. (1974). Studies of Rich Clusters of Galaxies I: Galaxy Counts for Nine Clusters of Intermediate Redshift and the Angular Diameter-Redshift Relation. *Monthly Notices of the Royal Astronomical Society* **167**, 437-456.
- Baade, W. (1952). A Revision of the Extra-Galactic Distance Scale. *Trans. IAU* **8**, 397-98.
- Baldwin, J. A., Burke, W. L., Gaskell, C. M. & Wampler, E. J. (1978). Relative quasar luminosities determined from emission line strengths. *Nature* **273**, 431-435.
- Balkenhol, L., et al. (2023). Measurement of the CMB temperature power spectrum and constraints on cosmology from the SPT-3G 2018 TT, TE, and EE dataset. *Physical Review D*, **108**, 023510 (35pp).
- Baum, W. A. (1957). Photoelectric determinations of redshifts beyond 0.2 c. *Astronomical Journal* **62**, 6-7.
- Baum, W. A. (1972). The Diameter-Redshift Relation. In D. S. Evans, D. Wills & B. J. Wills (eds.) *External Galaxies and Quasi-Stellar Objects*, Proceedings from IAU Symposium no. 44 (Dordrecht, Reidel), 393-396.
- Betoule, M. E. A. et al. (2014). Improved cosmological constraints from a joint analysis of the SDSS-II and SNLS supernova samples. *Astronomy & Astrophysics* **568**, A22 (32pp).
- Bigot, G., Fliche, H. H. & Triay, R. (1988). A determination of q_0 by a technique of null correlation. *Astronomy and Astrophysics* **206**, 1-6.
- Bode, M. F. & Evans, A. (1979). Visibility of pregalactic fluctuations and an upper limit on q_0 . *Astronomy and Astrophysics* **78**, 78-81.
- Bondi, H. & Gold, T. (1948). The steady-state theory of the expanding universe. *Monthly Notices of the Royal Astronomical Society* **108**, 252-270.
- Brown, G. S. & Tinsley, B. M. (1974). Galaxy counts as a cosmological test. *Astrophysical Journal* **194**, 555-558.
- Bruzual, A. G. & Spinrad, H. (1978). The characteristic size of clusters of galaxies-A metric rod used for a determination of the deceleration parameter. *Astrophysical Journal* **220**, 1-7.
- Camarena, D. & Marra, V. (2020). Local determination of the Hubble constant and the deceleration parameter. *Physical Review Research* **2**, 013028 (8pp).
- Chitre, S. M. & Narlikar, J. V. (1976). The effect of intergalactic dust on the measurement of the cosmological deceleration parameter q_0 . *Astrophysics and Space Science* **44**, 101-105.
- Chodos, A. & Detweiler, S. (1980). Where has the fifth dimension gone? *Physical Review D* **21**, 2167-2170.
- Ćirković, M. M. & Samurović, S. (2001). Cosmological constant and the fate of the DDM theory. *Astronomy & Astrophysics*, **373**, 377-380.
- Ćirković, M. M. & Perović, S. (2018). Alternative explanations of the cosmic microwave background: A historical and an epistemological perspective. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* **62**, 1-18.
- Coles, P. & Ellis, G. (1994). The case for an open Universe. *Nature* **370**, 609-615.
- Coles, P. & Lucchin, F. (1995). *Cosmology: The origin and evolution of cosmic structure* (John Wiley & Sons, New York).

- Dabrowski, M. & Stelmach, J. (1986). A redshift-magnitude formula for the universe with cosmological constant and radiation pressure. *Astronomical Journal* **92**, 1272-1277.
- Davidson, A., Hartig, G. & Fastie, W. (1977). Ultraviolet spectrum of quasi-stellar object 3C273. *Nature* **269**, 203-206.
- Davidson, W. (1962). Philosophical aspects of Cosmology. *British Journal for the Philosophy of Science* **13**, 120-129.
- Davidson, W. (1982). Some cosmological implications of Dirac's large numbers hypothesis. *Monthly Notices of the Royal Astronomical Society* **200**, 645-660.
- De Leon, J. P. (2006). An analytical model for the transition from decelerated to accelerated cosmic expansion. *International Journal of Modern Physics D* **15**, 1237-1257.
- de Sitter, W. (1917). On Einstein's theory of gravitation and its astronomical consequences. *Mon. Not. Roy. Astron. Soc.* **78**, 3-28.
- Di Valentino, E. (2021). A combined analysis of the H_0 late time direct measurements and the impact on the Dark Energy sector. *Monthly Notices of the Royal Astronomical Society* **502**, 2065-2073.
- Dick, S. (2015). *Discovery and Classification in Science: Controversy and Consensus* (Cambridge: Cambridge University Press).
- Dirac, P. A. M. (1974). Cosmological models and the large numbers hypothesis. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences* **338**, 439-446.
- Dressler, A. (1987). The Large-Scale Streaming of Galaxies. *Scientific American* **257**, 46-55.
- Dressler, A., Faber, S. M., Burstein, D., Davies, R. L., Lynden-Bell, D., Terlevich, R. J. & Wegner, G. (1987). Spectroscopy and photometry of elliptical galaxies-A large-scale streaming motion in the local universe. *Astrophysical Journal* **313**, L37-L42.
- D'Souza, R. & Bell, E. F. (2018). The Andromeda galaxy's most important merger about 2 billion years ago as M32's likely progenitor. *Nature Astronomy* **2**, 737-743.
- Duhem, P. (1906). *La Théorie Physique. Son Objet, sa Structure* (Chevalier & Rivière, Paris).
- Eastmond, T. S. (1965). The Deceleration Parameter, Stellar Evolution, and the Cosmological Constant. *Publications of the Astronomical Society of the Pacific* **77**, 385-387.
- Elgarøy, Ø., Mota, D. F. & Multamäki, T. (2005). Loitering universe models in light of the CMB. *Physical Review D* **71**, 067301.
- Ellis, G. F. R., Sievers, A. W., & Perry, J. J. (1984). Cosmological observations of galaxies - The observational map. *Astronomical Journal* **89**, 1124-1154.
- Einstein, A. (1917). Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie. *Sitz. König. Preuss. Akad.* 142-152.
- Einstein, A. & de Sitter, W. (1932). On the Relation between the Expansion and the Mean Density of the Universe. *Proceedings of the National Academy of Sciences* **18**, 213-214.
- Fabbri, R., Melchiorri, F., Mencaraglia, F. & Natale, V. (1979). The cosmological deceleration parameter and the Sunyaev-Zeldovich effect. *Astronomy and Astrophysics* **74**, L20-L23.
- Fang, L. (2016). *The most wanted man in China: My journey from scientist to enemy of the state* (P. Link, Trans.) (Henry Holt and Company, New York).
- Fang, L. Z., Zhou, Y. Y., Cheng, F. Z. & Chu, Y. Q. (1976). A statistical analysis for quasars with radio component structure. I. Redshift-magnitude relation. *Acta Astronomica Sinica* **17**, 134-146.
- Fang, L. Z., Kiang, T., Cheng, F. H. & Hu, F. X. (1982). Determinations of the Deceleration Parameter q_0 . *Quarterly Journal of the Royal Astronomical Society* **23**, 363-387.
- Freedman, W. L. (2021). Measurements of the Hubble constant: tensions in perspective. *The Astrophysical Journal* **919**, 16 (22pp).
- Freedman, W. L. & Madore, B. F. (2010). The Hubble constant. *Annual Review of Astronomy and Astrophysics* **48**, 673-710.
- Gehren, T., Fried, J., Wehinger, P. A. & Wyckoff, S. (1984). Host galaxies of quasars and their association with galaxy clusters. *Astrophysical Journal* **278**, 11-27.
- Godfrey-Smith, P. (2003). *Theory and reality: An introduction to the philosophy of science* (Chicago: University of Chicago Press).
- Gong, Y. & Wang, A. (2006). Observational constraints on the acceleration of the Universe. *Physical Review D* **73**, 083506 (6pp).
- Gott III, J. R. & Gunn, J. E. (1971). The coma cluster as an x-ray source: Some cosmological implications. *Astrophysical Journal* **169**, L13-L15.
- Gott III, J. R., Gunn, J. E., Schramm, D. N., & Tinsley, B. M. (1974). Unbound Universe. *Astrophysical Journal* **194**, 543-553.
- Gott III, J. R., Park, M. G., & Lee, H. M. (1989). Setting limits on q_0 from gravitational lensing. *Astrophysical Journal, Part 1 (ISSN 0004-637X)*, vol. 338, March 1, 1989, p. 1-12., 338, 1-12.
- Gudehus, D. H. (1972). Cosmology and information. *Publications of the Astronomical Society of the Pacific* **84**, 818-822.
- Guiderdoni, B. & Rocca-Volmerange, B. (1990). Constraints on the evolution of high-redshift galaxies and on $q(0)$ from faint galaxy counts. *Astronomy and Astrophysics* **227**, 362-378.
- Gunn, J. E. & Oke, J. B. (1975). Spectrophotometry of faint cluster galaxies and the Hubble diagram-an approach to cosmology. *Astrophysical Journal* **195**, 255-268.
- Gunn, J. E. & Tinsley, B. M. (1975). An accelerating universe. *Nature* **257**, 454-457.
- Guy, J., et al. (2010). The Supernova Legacy Survey 3-year sample: Type Ia supernovae photometric distances and cosmological constraints. *Astronomy & Astrophysics* **523**, A7 (34pp).

- Haggerty, M. J. & Wertz, J. R. (1972). On the redshift–magnitude relation in hierarchical cosmologies. *Monthly Notices of the Royal Astronomical Society* **155**, 495–503.
- Hickson, P. (1977). The angular-size-redshift relation. II-A test for the deceleration parameter. *Astrophysical Journal* **217**, 964–975.
- Hoyle, F. (1959). The relation of radio astronomy to cosmology. In *Symposium-International Astronomical Union, Paris Symposium on Radio Astronomy*, vol. **9** (Cambridge University Press, Cambridge), 529–532.
- Hoyle, F. & Sandage, A. (1956). The second-order term in the redshift-magnitude relation. *Publications of the Astronomical Society of the Pacific* **68**, 301–307.
- Hu, F. X. (1982). The Hubble diagram of quasars and the deceleration parameter q_0 . *Chinese Astronomy and Astrophysics* **6**, 93–97.
- Humason, M. L., Mayall, N. U. & Sandage, A. R. (1956). Redshifts and magnitudes of extra-galactic nebulae. *Astronomical Journal* **61**, 97–162.
- Islam, J. N. (2004). *An Introduction to Mathematical Cosmology* (2nd edition, Cambridge University Press, Cambridge).
- Jackson, N. (2015). The Hubble constant. *Living reviews in relativity* **18**, 2 (51pp).
- Janis, A. I. (1986). Angular size in an expanding universe. *American Journal of Physics* **54**, 1008–1011.
- Kamionkowski, M. & Riess, A. G. (2023). The Hubble tension and early dark energy. *Annual Review of Nuclear and Particle Science* **73**, 153–180.
- Kantowski, R. (1969). Corrections in the Luminosity-Redshift relations of the homogeneous Friedmann models. *Astrophysical Journal* **155**, 89–103.
- Kapahi, V. K., Kulkarni, V. K., & Subrahmanya, C. R. (1987). On the interpretation of the observed angular-size-flux-density relation for extragalactic radio sources. *Journal of Astrophysics and Astronomy* **8**, 33–50.
- Kardashev, N. (1967). Lemaitre's Universe and Observations. *Astrophysical Journal* **150**, L135–L139.
- Kellermann, K. I. (1993). The cosmological deceleration parameter estimated from the angular-size/redshift relation for compact radio sources. *Nature* **361**, 134–136.
- Kiang, T. & Cheng, F. H. (1982). Posterior probability of the deceleration parameter q_0 from quasars provided with a luminosity indicator. *Journal of Astrophysics and Astronomy* **3**, 363–378.
- Klapdor, H. V. & Grotz, K. (1986). Evidence for a nonvanishing energy density of the vacuum (or cosmological constant)? *The Astrophysical Journal* **301**, L39–L43.
- Kolb, E. W. (1989). A coasting cosmology. *Astrophysical Journal* **344**, 543–550.
- Kolb, E. W. & Turner, M. S. (1990). *The Early Universe* (Addison-Wesley, Boston).
- Kollerstrom, J. & McVittie, G. C. (1975). The redshift-distance relationship derived from clusters of galaxies. *The Observatory* **95**, 90–97.
- Kragh, H. 1996, *Cosmology and Controversy*. Princeton: Princeton University Press.
- Kragh, H. 2007, *Conceptions of Cosmos*. Oxford: Oxford University Press.
- Kragh, H. & Longair, M. (Eds.). (2019). *The Oxford Handbook of the History of Modern Cosmology*. Oxford: Oxford University Press.
- Kristian, J., Sandage, A., & Westphal, J. A. (1978). The extension of the Hubble diagram. II-New redshifts and photometry of very distant galaxy clusters-First indication of a deviation of the Hubble diagram from a straight line. *Astrophysical Journal* **221**, 383–394.
- Krolik, J. H. & Raymond, J. C. (1988). Measuring H_0 and q_0 with X-ray lines from galaxy clusters. *Astrophysical Journal* **335**, L39–L42.
- Lacey, C. & Cole, S. (1993). Merger rates in hierarchical models of galaxy formation. *Monthly Notices of the Royal Astronomical Society* **262**, 627–649.
- Layzer, D. (1962). Galaxy Clusters and Cosmology. *Astrophysical Journal* **136**, 138–149.
- Lemaître, G. (1931). The expanding universe. *Mon. Not. Roy. Astron. Soc.* **91**, 490–501.
- Liang, E., & Zhang, B. (2005). Model-independent multivariable gamma-ray burst luminosity indicator and its possible cosmological implications. *The Astrophysical Journal* **633**, 611–623.
- Linder, E. V. (1988). Cosmological tests of generalized Friedmann models. *Astronomy and Astrophysics* **206**, 175–189.
- Longair, M. S. (1971). Observational cosmology. *Reports on Progress in Physics*, **34**(3), 1125.
- Maudlin, T. 2007, *The Metaphysics Within Physics* (Oxford University Press, Oxford).
- Mattig, W. (1957). Über den Zusammenhang zwischen Rotverschiebung und scheinbarer Helligkeit. *Astronomische Nachrichten* **284**, 109–111.
- McCrea, W. H. & Milne, E. A. (1934). Newtonian universes and the curvature of space. *Quarterly Journal of Mathematics* **5**, 73–80.
- McIntosh, C. B. G. (1968). Cosmological models with both radiation and matter. *Monthly Notices of the Royal Astronomical Society* **140**, 461–472.
- Meinel, R. (1981). Intergalactic extinction and the deceleration parameter. *Astronomische Nachrichten* **302**, 177–181.
- Menci, N., Adil, S. A., Mukhopadhyay, U., Sen, A. A. & Vagnozzi, S., 2024. Negative cosmological constant in the dark energy sector: tests from JWST photometric and spectroscopic observations of high-redshift galaxies. *Journal of Cosmology and Astroparticle Physics* **7**, 072 (34pp).
- Milne, E. A. 1935, *Relativity, Gravitation and World Structure* (Oxford University Press, Oxford).
- Neben, A. R. & Turner, M. S. (2013). Beyond H_0 and q_0 : Cosmology is no longer just two numbers. *The Astrophysical Journal* **769**, 133 (8pp).

- Newberg, H. J. M. (1992). *Measuring q_0 using supernovae at z approximately equal to 0.3*. Ph.D. dissertation. University of California, Berkeley.
- Noonan, T. W. (1972). A Suggested Concept for Galaxy-Cluster Size for Use in the Angular-Size Cosmological Test. *Astronomical Journal* **77**, 134-137.
- O’Raifeartaigh, C., O’Keeffe, M., Nahm, W., & Mitton, S. (2015). Einstein’s cosmology review of 1933: a new perspective on the Einstein-de Sitter model of the cosmos. *The European Physical Journal H* **40**, 301-335.
- Ostriker, J. P. & Steinhardt, P. J. (1995). The observational case for a low-density Universe with a non-zero cosmological constant. *Nature* **377**, 600-602.
- Ott, H. A. (2005). Galaxy redshift-number counts with MRSP data: A method of estimating q_0 . In *Large-Scale Structures in the Universe Observational and Analytical Methods* (Springer, Berlin), 274-281.
- Peach, J. V. (1970). The determination of the deceleration parameter and the cosmological constant from the redshift-magnitude relation. *Astrophysical Journal* **159**, 753-763.
- Peacock, J. A. (1999). *Cosmological physics*. Cambridge: Cambridge University Press.
- Peebles, P. J. E. (1993). *Principles of physical cosmology*. Princeton: Princeton University Press.
- Peebles, P. J. E., Page, L. A. Jr. & Partridge, R. B. (2009). *Finding the Big Bang*. Cambridge: Cambridge University Press.
- Pen, U. L. (1997). Measuring the universal deceleration using angular diameter distances to clusters of galaxies. *New Astronomy* **2**, 309-317.
- Perlmutter, S. et al. (1996). High-redshift supernova discoveries on demand: First results from a new tool for cosmology and bounds on q_0 . *Nuclear Physics B-Proceedings Supplements* **51**, 20-29.
- Perlmutter, S., et al. (1998). Discovery of a supernova explosion at half the age of the universe. *Nature*, **391**, 51-54.
- Perlmutter S. et al. (1999). Measurement of Ω and Λ from 42 high-redshift supernovae. *Astrophys. J.* **517**, 565-586.
- Perović, S. & Ćirković, M. M. (2024). *The Cosmic Microwave Background: Historical and Philosophical Lessons*. Cambridge: Cambridge University Press.
- Petrosian, V. (1970). Redshift Distribution of Quasistellar Radio Sources. *Astrophysical Letters* **6**, 71.
- Phillipps, S. (1983). Limits on the accuracy of determining $q(0)$ from supernovae. *Astrophysical Letters* **23**, 145-149.
- Phillipps, S. & Ellis, R. S. (1983). The distribution of absorption lines in QSO spectra. *Monthly Notices of the Royal Astronomical Society* **204**, 493-506.
- Raine, D. J. & Thomas, E. G. (1981). Large-scale inhomogeneity in the Universe and the anisotropy of the microwave background. *Monthly Notices of the Royal Astronomical Society* **195**, 649-660.
- Raychaudhuri, A. K. & Mukherjee, G. (1984). Is the Universe near the state of maximal expansion? *Monthly Notices of the Royal Astronomical Society* **209**, 353-358.
- Rines, K., Forman, W., Pen, U., Jones, C. & Burg, R. (1999). Constraining q_0 with cluster gas mass fractions: A feasibility study. *The Astrophysical Journal* **517**, 70-77.
- Sahni, V., Feldman, H. & Stebbins, A. (1992). Loitering universe. *Astrophysical Journal* **385**, 1-8.
- Salim, S. (2014). Green Valley Galaxies. *Serbian Astronomical Journal* **189**, 1-14.
- Sandage, A. (1961). The Ability of the 200-inch Telescope to Discriminate Between Selected World Models. *Astrophysical Journal* **133**, 355-392.
- Sandage, A. (1962). The Change of Redshift and Apparent Luminosity of Galaxies due to the Deceleration of Selected Expanding Universes. *Astrophysical Journal* **136**, 319-333.
- Sandage, A. (1968). Observational cosmology. *The Observatory* **88**, 91-106.
- Sandage, A. R. (1970). Cosmology: A search for two numbers. *Physics Today* **23**, 34-41.
- Sandage, A. R. (1971). The age of the galaxies and globular clusters: Problems of finding the Hubble constant and deceleration parameter. In *Study Week on Nuclei of Galaxies* (p. 601-632).
- Sandage, A. R. (1972). The redshift-distance relation. II. The Hubble diagram and its scatter for first-ranked cluster galaxies: a formal value for q_0 . *The Astrophysical Journal* **178**, 1-24.
- Sandage, A. & Tammann, G. A. (1975). Steps toward the Hubble constant. V-The Hubble constant from nearby galaxies and the regularity of the local velocity field. *Astrophysical Journal* **196**, 313-328.
- Sasaki, S. (1996). A new method to estimate cosmological parameters using the baryon fraction of clusters of galaxies. *Publications of the Astronomical Society of Japan* **48**, L119-L122.
- Schrabback, T., et al. (2010). Evidence of the accelerated expansion of the Universe from weak lensing tomography with COSMOS. *Astronomy & Astrophysics* **516**, A63 (26pp).
- Schuecker, P., et al. (1998). The Muenster Redshift Project. III. Observational constraints on the deceleration parameter. *The Astrophysical Journal* **496**, 635-647.
- Shanks, T., Stevenson, P. R. F., Fong, R. & MacGillivray, H. T. (1984). Galaxy number counts and cosmology. *Monthly Notices of the Royal Astronomical Society* **206**, 767-800.
- Silk, J. (1974). Large-scale inhomogeneity of the universe: implications for the deceleration parameter. *Astrophysical Journal* **193**, 525-528.
- Solheim, J. E. (1966). Relativistic World Models and Redshift-Magnitude Observations. *Monthly Notices of the Royal Astronomical Society* **133**, 321-328.
- Sorrell, W. H. (2008). The cosmic microwave background radiation in a non-expanding universe. *Astrophysics and Space Science* **317**, 59-70.
- Stabell, R. & Refsdal, S. (1966). Classification of general relativistic world models. *Monthly Notices of the Royal Astronomical Society* **132**, 379-388.

- Stepanas, P. G., & Saha, P. (1995). Estimating q_0 from angular size statistics. *Monthly Notices of the Royal Astronomical Society* **272**, L13-L15.
- Stockton, A. (1978). The nature of QSO redshifts. *Astrophysical Journal* **223**, 747-757.
- Stoelinga, S. J. M. (1985). On the cosmological parameters. *Astrophysical Journal* **291**, 396-398.
- Tammann, G. A. (1974). The Hubble constant and the deceleration parameter. In *Symposium-International Astronomical Union* **63** (Cambridge University Press, Cambridge), 47-59.
- Tinsley, B. M. (1972). A First Approximation to the Effect of Evolution on q_0 . *Astrophysical Journal* **173**, L93-L97.
- Tomita, K. & Hayashi, C. (1963). The cosmical constant and the age of the universe. *Progress of Theoretical Physics* **30**, 691-699.
- Trimble, V. (1996). H_0 : The incredible shrinking constant, 1925-1975. *Publications of the Astronomical Society of the Pacific* **108**, 1073-1082.
- Turner, E. L. (1979). Statistics of the Hubble diagram. I. Determination of q_0 and luminosity evolution with application to quasars. *Astrophysical Journal* **230**, 291-303.
- Turner, M. S. (2018). Λ CDM: Much more than we expected, but now less than what we want. *Foundations of Physics* **48**, 1261-1278.
- Wagoner, R. V. (1977). Determining q_0 from supernovae. *Astrophysical Journal* **214**, L5-L7.
- Whitford, A. E. (1971). Absolute energy curves and K-corrections for giant elliptical galaxies. *The Astrophysical Journal* **169**, 215-223.
- Wiik, K. & Valtaoja, E. (2001). The geometry of the universe from high resolution VLBI data of AGN shocks. *Astronomy & Astrophysics* **366**, 1061-1064.
- Xie, G., et al. (2001). The Hubble diagram of metal-rich QSOs and deceleration parameter q_0 . *Sci. China Ser. A-Math.* **44**, 1081-1088 (2001).
- Xu, L. & Liu, H. (2008). Constraints to deceleration parameters by recent cosmic observations. *Modern Physics Letters A* **23**, 1939-1948.
- Yoshii, Y. & Takahara, F. (1989). On the redshift-volume measurement of the cosmological density parameter. *Astrophysical Journal* **346**, 28-33.
- Zhao, G.-B., Raveri, M., Pogosian, L., Wang, Y., Crittenden, R. G., Handley, W. J., Percival, W. J., Beutler, F., Brinkmann, J. & Chuang, C.-H. et al. (2017). Dynamical dark energy in light of the latest observations. *Nature Astronomy* **1**, 627-632.
- Zotov, N. V. & Davidson, W. (1970). An analysis of the redshift-apparent magnitude relation for 147 quasi-stellar objects. *Australian Journal of Physics* **23**, 351-362.