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- Traditionally, investment is defined as the current commitment of resources in order to achieve later benefits.

- If resources and benefits take the form of money, investment is the present commitment of money for the purpose of receiving (hopefully more) money later.
- The money to be obtained later is, in some cases, known exactly, but, in most situations, this amount is uncertain.

- A broader viewpoint, based on the idea of flows of expenditures and receipts spanning a period of time.

- From this viewpoint, the objective of investment is to tailor the pattern of these flows over time to be as desirable as possible.

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- If these flows are denominated in cash, we have cash flows, and the series of flows over several periods is termed a cash flow stream.

- Example: By taking out a loan, it may be possible to exchange a large negative cash flow next month for a series of smaller negative cash flows over several months, and this alternative cash flow stream may be preferred to the original one.

- Investment science is the application of scientific tools to investments. The scientific tools used are primarily mathematical.

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- Usually the cash flows (either positive or negative) occur at known specific dates, such as at the end of each quarter of a year or at the end of each year. The stream can then be described by listing the flow at each of these times.

- Example: If the basic time period is taken as one year, one possible stream over a year, from beginning to end is (-1, 1.2), corresponding to an initial payment of \$1 at the beginning of the year and the receipt of \$1.20 a year later.

- Example: An investment over four years might be (-1, 0.10, 0.10, 0.10, 1.10), where an initial investment of \$1 leads to payment of \$0.10 at the end of each year for three years and then a final payment of \$1.10

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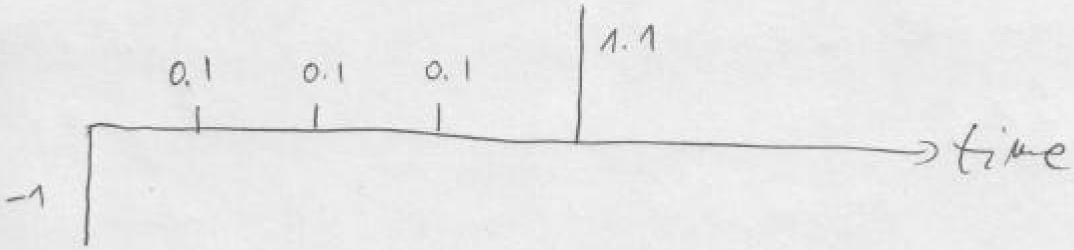
- Cash flow streams can also be represented in diagram form:



- If the magnitudes of some future cash flows are uncertain, a more complex representation of a cash flow stream must be employed.

- Typical questions of investments: Which of two cash flow streams is most preferable? How much would I be willing to pay to own a given stream? If I can purchase a share of a stream, how much should I purchase? Given a collection of available cash flow streams, what is the most favourable combination of them?

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- Important principles are governing the market and investments science:

1) The Comparison principle
2) No Arbitrage principle
3) Dynamics
4) Risk Aversion

- One way that the risk aversion principle is formalized is through mean-variance analysis. In this approach, the uncertainty of the return on an asset is characterized by just two quantities: the mean value of the return and the variance of the return. The risk aversion principle then says that if several investment opportunities have the same mean but different variances, a rational (risk-averse) investor will select the one that has the smallest variance.

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— Typical Investment Problems:

1) Pricing

Imagine that there is an investment opportunity that will pay exactly $\$110 = X$ at the end of one year. We ask! How much is this investment worth today? I.e., what is the appropriate price of this investment? $X/(1+r)$

2) Hedging

Process of reducing the financial risks that either arise in the course of normal business operations or are associated with investments. Example: Insurance. By paying a fixed amount (a premium), you can protect yourself against certain specified possible losses, such as losses due to fire, theft or even adverse price changes.

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3) Pure Investment

Portfolio selection Problem

etc.

— Deterministic Cash Flow Streams

— Simple interest rule: Money invested in a bank account (amount of money equal to A) that pays interest r per year will give you in your bank account at the end of one year the <u>principal</u> ($=A$) plus <u>interest</u> ($=rA$); total will be $(1+r)A$.

— Money invested for a period different from one year accumulates interest proportional to the total time of the investment. The total value V after n years is

$$V = (1 + rn)A$$

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- If the proportional rule holds for fractional years, then after any time t (measured in years), the account value is
$$V = (1+rt)A$$

e.g. t can be $t = 1.2$ years etc.

The account grows linearly with time

- Compound interest: Most bank accounts and loans employ some form of compounding: If interest is compounded yearly, then after one year, the first year's interest is added to the original principal to define a larger principal base for the second year. So, during the second year, the account earns interest on interest — this is the compounding effect.
- Under yearly compounding, after n years
$$V = (1+r)^n A.$$

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- Seven-Ten rule: Money invested at 7% per year doubles in approximately 10 years. Also, money invested at 10% per year doubles in approximately 7 years.

- Compounding at Various Intervals:
Most banks now calculate and pay interest more frequently — quarterly, monthly, or even daily. It is traditional to still quote the interest rate on a yearly basis, but then apply the appropriate proportion of that interest rate over each compounding period. If a year is divided into a fixed number of equally spaced periods — say m periods, then the account grows by $1+\frac{r}{m}$ during one period. After k periods we have $V=\left(1+\frac{r}{m}\right)^{k}A$, and after a year,

$$V=\left(1+\frac{r}{m}\right)^{m}.$$

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- Effectively, the interest rate r' is

$$1+r' = \left(1+\frac{r}{m}\right)^m$$

- Continuous compounding: We can divide the year into smaller and smaller periods. This leads into

$$V = \lim_{m\to\infty} \left(1+\frac{r}{m}\right)^m A = e^{r} \cdot A$$

- In this case: $t = \frac{K}{m} \Rightarrow$

$$\left(1+\frac{r}{m}\right)^K = \left(1+\frac{r}{m}\right)^{mt} = e^{rt},$$ so

$$V(t) = e^{rt} A$$

- The same applies to debts.

- The money invested today leads to increased value in the future as a result of interest. This set of formulas can be reversed in time to calculate that should be assigned now, in the present,

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to money that is to be received later.

- Consider two situations: (1) you will receive \$110 in 1 year, (2) you receive \$100 now and deposit it in a bank account for 1 year at ~~[illegible]~~ 10% interest. Clearly, these situations are identical after 1 year.

- We can restate this equivalence by saying that \$110 received in 1 year has a <u>present value</u> of \$100. In general, amount of money A received after 1 year has a present value $A/(1+r)$. Or, more generally, present value of a money received after k periods of time with the m-compounding (m periods is equal to 1 year) is

$$d_k \cdot A, \text{ where } d_k = \frac{1}{\left(1+\frac{r}{m}\right)^k}$$

is a discount factor. The whole procedure is called discounting.