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- Internal Rate of Return (IRR): Given a cash flow stream $(x_0, x_1, \dots, x_n)$ associated with an investment, the present value is

$$PV = \sum_{k=0}^{n} \frac{x_k}{(1+r)^k}$$ (assuming yearly compounding at the rate r)

IRR is a rate r for which $PV = 0$.

It satisfies the equation

$$0 = x_0 + \frac{x_1}{1+r} + \frac{x_2}{(1+r)^2} + \dots + \frac{x_n}{(1+r)^n}$$

If we introduce $c = \frac{1}{1+r}$, the equation becomes

$$0 = x_0 + x_1 c + x_2 c^2 + \dots + x_n c^n,$$

and $r = \frac{1}{c} - 1$.

- Example: For a cash flow stream $(-2, 1, 1)$ we have $-2 + c + c^2 = 0 \Rightarrow c_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} \Rightarrow c_1 = 1, \; c_2 = -2 \Rightarrow r_1 = \frac{1}{c_1} - 1 = 0, \; r_2 = \frac{1}{c_2} - 1 = -\frac{3}{2}$.

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- Note that the IRR is defined without reference to a prevailing interest rate from the outside financial world. It is determined entirely by the cash flow stream.

- IRR is the rate of return that an ideal bank would have to apply so that PV of the given cash flow stream is equal to zero.

- IRR can be defined also for all other types of compounding.

- Example: For a cash flow stream (-2, 1, 1, 1) we can calculate the IRR by using Excel. The result is IRR = 23%.

- Evaluation criteria:
The essence of investment is selection from a number of alternative cash flow streams. Several different criteria are used in practice. We will discuss the two most important methods.

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- Method 1: Net Present Value (NPV)

This method evaluates alternatives by simply ranking them according to their present values — the higher the PV, the more desirable the alternative.

- To emphasize that we are including all cash flows (positive as well as negative ones), we will use the term Net Present Value (NPV) here. We will also use terms Present worth of benefits and Present worth of costs, both of which are just PVs of positive cash flows and negative cash flows. NPV is the difference between these two terms (Present worth of costs is, by definition, positive number, absolute value of the corresponding PV).

- Example: Suppose that you have the opportunity to plant trees that later can be sold for lumber. This project requires

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an initial outlay of money in order to purchase and plant the seedlings. No other cash flow occurs until the trees are harvested. However, you have a choice as to when to harvest: after one year or after two years. The cash flow streams associated with these two alternatives are:

(a) (−1, 2) cut early (after one year)

(b) (−1, 0, 3) cut later (after two years)

If we assume that the prevailing rate is 10%, then NPVs are

(a) $NPV = -1 + \frac{2}{1.1} = 0.82$

(B) $NPV = -1 + \frac{3}{1.12} = 1.48.$

According to NPV criterion, it is best to cut later.

— This criterion can be used for combination of several investment (even of different duration, since NPV can be calculated for all of them and then, summed)

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- Method 2: Internal Rate of Return (IRR)

IRR can also be used to rank alternative cash flow streams: the higher the IRR, the more desirable the investment. However, a potential investment, or project, is presumably not worth considering unless its IRR is greater than the prevailing interest rate.

- Example: When to cut a tree?

(a) For $c = \frac{1}{1+r}$ the equation is

$$-1+2c=0 \Rightarrow c=\frac{1}{2} \Rightarrow r=\frac{1}{c}-1=1$$

(b) The equation is

$$-1+3c^2=0 \Rightarrow c=\frac{1}{\sqrt{3}} \Rightarrow r=\frac{1}{c}-1=\sqrt{3}-1\approx 0.73.$$

The best alternative according to this criterion is to <u>cut early.</u>

- Note that the conclusion of the above exaple is opposite to the conclusion obtained according to the NPV criterion.

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—Discussion of the criteria:

- there is considerable debate as to which of the two criteria, NPV of IRR, is the most appropriate for investment evaluation. NPV is simplest to calculate, IRR can have several solutions. But IRR depend only on cash flow stream, not on the outside financial word, which can change.
- The primary difference between the two criteria can be explained in terms of the "When to cut a tree" example. We must think of this process as a series of cycles. Suppose that the proceeds of the first harvest are used to plant additional trees, starting a long series of expansion in the tree forming business. Under plan (a), cut early, the business can be doubled every year. In plan (b), cut later, the business can be tripled every two years.

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- Tripling every 2 years is equivalent, in the long run, to increasing by a factor of $\sqrt{3}$ every year. The yearly growth rates of these two plans, factors 2 and $\sqrt{3}$, respectively, are each equal to 1 plus the corresponding IRR – and this is valid in general.
- So, in this kind of situation, where the proceeds of the investment can be repeatedly reinvested in the same type of project but scaled in size, it makes sense to select the project according to IRR criterion.
- On the other hand, if the project is one-time opportunity and cannot be repeated, NPV is the criterion of choice.
- Remark: NPV is prefered by theorist. It can be improved so that its results are in consistence with the IRR criterion.

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-Homework X:

1) A major lottery advertises that it pays the winner $10 milion. However, this prize money is paid at the rate of $500,000 each year (with the first payment being immediate) for a total of 20 payments. What is the PV of this prize at 10% interest rate?

2) IHQF/CFP student is inquisitive and determined to learn both theory and the application of investment theory. He/she pressed the tree farmer (from the example) for additional information and learned that it was possible to delay cutting the trees for another year. The farmer said that, from a PV perspective, it was not worthwile to do so. The student instantly deduced that the revenue obtained must be less than x. What is x?