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A Comparison of Practical Systems for Producing a Uniform Magnetic Field for Electron Scattering Experiments

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Abstract.
This paper presents a comparison of the system used in our Stepwise Electron Laser Excitation experiment with other practical systems for producing a uniform magnetic field. The need to use both analytical and numerical methods for analysis of magnetic field uniformity is shown.

Keywords: Electron Scattering, Trochoidal monochromator, uniform magnetic field
PACS: 34.80-i, 41.85.-Lc, 85.70.-w

INTRODUCTION

In our Stepwise Electron Laser Excitation (SELE) experiment the cylindrical trochoidal electron monochromator [1] is utilized. Crossed magnetic and electric field in cylindrical symmetry are used to select the electrons. For this kind of monochromator, the uniform magnetic field is one of the basic requirements. Because of the large range in which uniform magnetic field is required in our experiment, the most common solution of using Helmholtz coil is abandoned. Our three-coil system is described in [2]. Meantime, we became aware of some results concerning characteristics of a system very similar to ours [3]. The system is similar to ours in a sense that it consists of three coils of equal radius. However, the models of systems are different, authors of [3] approximate coil as a current loop, our system is modeled as a real physical windings, using numerical integration. We shortly review the analytical approach and compare the results, obtained in this way [3], with ours, obtained numerically. Then, we discus the possibilties of practical realization of a five-coil system.

THE THREE-COIL SYSTEM

For the system of three coils, let R be the radius of each circle, I be electric current through the coils connected in series, the number of turns of the outside coil be equal to N, kN be the number of turns of the central coil (where k is a proportional coefficient to be determined), and a be the distance from the outside circles to the central one. To alleviate stability problems, the same current should supply all coils connected in series. Then the magnetic field intensity at any point on the axis for the three-coil system is

$$B(z) = \frac{\mu_0}{2}NR^2I\{\left[R^2+(a+z)^2\right]^{-3/2} + \left[R^2+(a-z)^2\right]^{-3/2} + (R^2+z^2)^{-3/2}\} \tag{1}$$

By symmetry of three-coil system, it is evident that $B(z)$ is an even function. Expanding $B(z)$ in the Taylor series, we can write

$$B(z) = B(0) + \frac{1}{2}B^{(2)}(0)z^2 + \frac{1}{24}B^{(4)}(0)z^4 + \frac{1}{726}B^{(6)}(0)z^6 + \cdots \tag{2}$$

where $B^{(2)}(0)$, $B^{(4)}(0)$, and $B^{(6)}(0)$ represent values of the second, fourth, and sixth derivatives of $B(z)$ at the center O, respectively.

From expression (2) it can be seen that the magnetic field on the axis is very close to homogeneous magnetic field if the conditions $B^{(2)}(0) = 0$ and $B^{(4)}(0) = 0$ are satisfied.

From expression (1) it could be derived

$$B^{(2)}(0) = \frac{3}{2}\mu_0 NR^2 I\left[\frac{2(4a^2-R^2)}{(R^2+a^2)^{7/2}} - \frac{k}{R^5}\right] \tag{3}$$

$$B^{(4)}(0) = \frac{1}{2}\mu_0 NR^2 I\left[\frac{1890a^4}{(R^2+a^2)^{11/2}} - \frac{1260a^2}{(R^2+a^2)^{9/2}} + \frac{90}{(R^2+a^2)^{7/2}} + \frac{45k}{R^7}\right] \tag{4}$$

By finding suitable values for a and k, uniformity of magnetic field is optimized. Setting the right-hand side of expressions (3) and (4) to zero, following results are obtained by numerical calculations [3]:

$$a/R = 0.7601 \text{ and } k = 59/111 = 0.5315$$

This results are slightly different from our results [2]:

$$a/R = 0.75 \text{ and } k = 0.5585$$

The reason is very simple: authors of [3] didn't take into account the physical dimensions of coils, they approximate coil as a current loop. We tried to put results presented in [3] ($a/R = 0.7601$ and $k = 59/111 = 0.5315$) into our simulation program [4] and the uniformity of magnetic field was significantly inferior compared to theoretically predicted results, presented in [3], and shortly reviewed in the next section. Our numerical method, proposed in [4] (in serbian), and in [2] (in english), is therefore more accurate.

COMPARISON OF UNIFORMITY OF MAGNETIC FIELD OF HELMHOLTZ COIL, MAXWELL'S TRICOIL, AND IMPROVED HELMHOLTZ COIL

Let us first recapitulate that for the Helmholtz coil the distance between the coils is equal to the radius of coils. For Maxwell's tricoil, let R be the radius of the great coil. Then, the radius of the each of small coils is R$\sqrt{4}/7$, the distance of each small coil from the great coil is R$\sqrt{3}/7$, the number of turns ratio between the great coil and the small ones is 64/49. The improved Helmholtz coil [3] is actually the system identical to ours.

The relative difference between magnetic field intensity at any point on the axis and the center O is:

$$\varepsilon = \frac{B(z) - B(0)}{B(0)}$$

where z denotes the distance on axis.

For Helmholtz coil, for $|\varepsilon| \leq 1\%$, $|z/R| \leq 0.3137$; for $|\varepsilon| \leq 0.1\%$, $|z/R| \leq 0.1731$.

For Maxwell'l tricoil, for $|\varepsilon| \leq 1\%$, $|z/R| \leq 0.4320$; for $|\varepsilon| \leq 0.1\%$, $|z/R| \leq 0.2864$.

For improved Helmholtz coil [3], and our system [2], for $|\varepsilon| \leq 1\%$, $|z/R| \leq 0.5104$; for $|\varepsilon| \leq 0.1\%$, $|z/R| \leq 0.3304$.

It is easy to see that, for the same value of relative difference ε, the axial uniform region is largest for improved Helmholtz coil and our system, then for Maxwell's tricoil, and the last, for the Helmholtz coil.

It should be noted that, for practical realizations, the measured results are nearer to predicted values for our system, than for [3]. Although convenient for analytical method, approximation of the windings as a current loop leads to an error, especially if the dimensions of the windings are not insignificant compared to the dimensions of the system.

Generally, analytical method is usefull to determine the initial conditions for solution, for example the location of windings. However, for fine tuning of system, simulations taking into account the physical dimensions of windings should be used.

THE DISCUSSION OF IMPROVED FIVE-COIL SYSTEM

The five-coil system, we presented in [2], solved the problem of overheating of the windings. However, this system is actually a three-coil case, since outer coils are placed one to another, as closely as possible. Technically, it was the simplest solution. In meantime, we ask ourselves a question: is it possible to improve uniformity of magnetic field by optimizing the locations of fourth and fifth coils, i.e. by realization of improved five-coil system. Of interest is a system where all five coils with circular windings would be of the same dimensions. Again, the coils should be connected in series, supplied with the same current. Preliminary simulations show that it is difficult to obtain optimal solution without the aid of analytical approach, namely, there are too many free parameters to optimize. The analytical approach should fix some of the parameters, presumably the locations of windings, and the number of turns ratios then will be easier and more accurate to determine using numerical solution. This question is still opened in literature.

CONCLUSION

This paper presented a comparison of practical systems for producing a uniform magnetic field convenient to use in our SELE experiment. After analysis of analytical method presented in [3], and by testing its results by our simulations, we concluded that our numerical method [4], is more accurate. The question of realization of five-coil system consisting of equal circular windings, to the best of our knowledge, is still opened in literature.

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