

A Visual Analysis of Calculation-Paths of the Mandelbrot Set

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Abstract: - In this paper we present a new method for analyzing some properties of the Mandelbrot Set. The algorithm used for our visual analysis is closely connected to Pickover Stalks and Buddhabrot method. Pickover Stalks method created biomorphs, diverse and complicated forms greatly resembling invertebrate organisms. Our method extends these previously developed methods that introduce the concept of preserving information about calculation steps when calculating the Mandelbrot Set. We create images that visualize statistical information for the calculation-paths of points tested for belonging to the Mandelbrot Set. Two variations of this method are presented: one that only takes into account the paths taken and one that also uses information about their lengths. We have developed software that enables us to analyze new properties of the Mandelbrot Set that can be seen when these new display methods are used. In this paper we also present in detail important features of our software.

Key-Words: - Mandelbrot, Buddhabrot, Fractal, Algorithm, Pickover Stalks

1 Introduction

Fractals are geometric shapes that have the property of self-similarity, or in other words, that the shape can be divided into parts that are reduced size copies (at least approximately) of the whole. Benoît Mandelbrot first used the term in 1975. The application and use of fractals has been increasing with the increase of computer power. They have shown their usability in a wide range of domains from biology and medicine [1], image processing [2], [3], art etc.

Fractal theory gives methods for describing the irregularity of natural objects, opposite to the idealizations created when using Euclidean geometry. The fractal dimension can be seen as a measure of complexity, or as an index of the scale-dependency of a pattern. This measure is defined mathematically with Hausdorff dimension [4].

Natural objects do not exhibit exact self-similarity, but to some degree statistical similarities. One direction of application of fractals in biology is calculating its fractal dimension and using it for a comparison between systems [5], [6]. The relevance of this parameter has been shown on the example of different sized insects living on a tree trunk and the distances they travel on it. If the bark has a fractal dimension of $D = 1.4$, an insect an order of magnitude smaller than another one perceives a

length increase of $10^{D-1} = 10^{0.4} = 2.51$, or a habitat surface area increase of $2.51^2 = 6.31$.

The second direction of the application of fractals in biology is in artificially creating biological objects or systems. An example is the use of iterated functions system (IFS) fractals for creating virtual trees [7]. C.A. Pickover demonstrated a new concept of Mandelbrot Set (M-set) [8] coloring that created images closely corresponding to single cellular organisms which were named biomorphs [9], some of these images can be seen in Fig. 1. This has shown the connection of the M-set and living organisms and the possible importance of researching its properties for biological science.

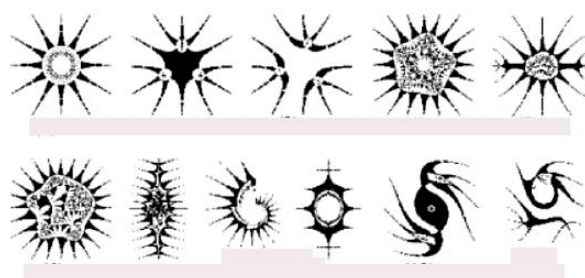


Figure 1. Examples of Biomorphs created using C.A. Pickover's technique

For the biomorphs creation, an essential step was the concept of Pickover Stalks. This was the first method that observes the behavior of points during

the calculation the M-Set; the coloring was based on how closely the orbits of interior points come to the x and y axes. A novel approach to displaying the M-Set is the Buddhabrot method, which uses information of the number of visits to points in the iterative creation algorithm [10]. We extend this method by preserving not only the information of which points have been visited, but also the order in which they have been visited. We visualize the paths points pass in the iterative process. To further explore and understand these images we have created fractal generator software, and we present its features in this paper.

The paper is organized as follows. In Section 2 we give the basic properties of the M-Set and corresponding Julia Sets (J-Sets). In Section 3 we show the algorithm for creating the M-Set and the Buddhabrot method. In Section 4 we present our extension of the Buddhabrot technique based on calculation paths. In the next section we analyze some characteristic images of M-Set created with the techniques presented. In the Section 6 we present the features of our fractal generator software.

2 M-Set and Basic Properties

The M-Set is defined as the set of complex values of c for which $|Z_n|$ under iteration of the complex quadratic polynomial $Z_{n+1} = Z_n^2 + c$ remains bounded. In other words, a complex number c is in the M-Set if, starting with $Z_0=0$ and using the iteration formula repeatedly, the absolute value of Z_n never becomes greater than a certain number however large the number of iterations.

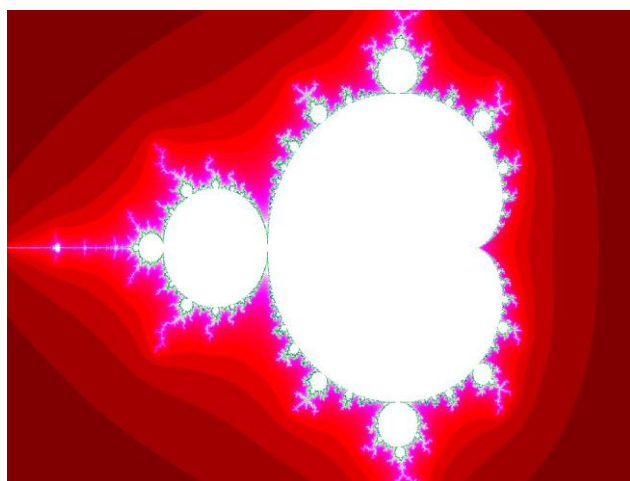


Figure 2. The Mandelbrot Set for function $Z_{n+1} = Z_n^2 + c$, maximum iterations 128, in the area $(-2, 1) \times (-i, i)$

It is formally defined with a set of quadric polynomials

$$\begin{aligned} P_c &: \mathbb{C} \rightarrow \mathbb{C} \\ P_c &: z \rightarrow z^2 + c \end{aligned} \quad (1)$$

If $P_c^n(z)$ presents the n -th iteration of $P_c(z)$. The Mandelbrot set is the subset M of the complex plane defined by Equation 2 (Fig. 2)

$$M = \{c \in \mathbb{C} : |\sup_{n \in \mathbb{N}} P_c^n(0)| < \infty\} \quad (2)$$

It has been shown that a complex number c is inside the M-Set if and only if $P_c^n(z) \leq 2$ for all n . It is a compact set, contained in the closed disk of radius 2 around the origin. The area of the M-set obtained by pixel counting was approximately 1.50659177 [11], and later by statistical sampling it has been shown that the area is 1.506484 with 95% confidence [12]. The area of the M-Set intersects the real axis x only in the area $[-2, 0.25]$. The parameters along this interval can be put in one-to-one correspondence with those of the real logistic family from equation [13],

$$z \rightarrow \lambda z(1-z), \quad \lambda \in [1, 4] \quad (3)$$

Where the connection between c and λ is the following

$$c = \frac{1 - (1 - \lambda)^2}{4} \quad (4)$$

The M-Set is a connected set. The boundary of the M-Set is called the Mandelbrot curve. It is bifurcation locus of the quadratic family defined in Equation 1, which means that the dynamical behavior changes drastically under small perturbation of the parameter c . The Hausdorff dimension of the boundary of the Mandelbrot set equals 2 as determined by a result of Mitsuhiro Shishikura [14].

There is a close correspondence of the Mandelbrot Set with Julia sets [15], [16]. A Julia set can also be defined with a set of quadric polynomials Equations 5, 6. (Fig. 3)

$$\begin{aligned} J_a &: \mathbb{C} \rightarrow \mathbb{C} \\ J_a &: z \rightarrow z^2 + a \end{aligned} \quad (5)$$

$$J_a = \{z \in \mathbb{C} : \sup_{n \in \mathbb{N}} J_a^n(z) < \infty\} \quad (6)$$

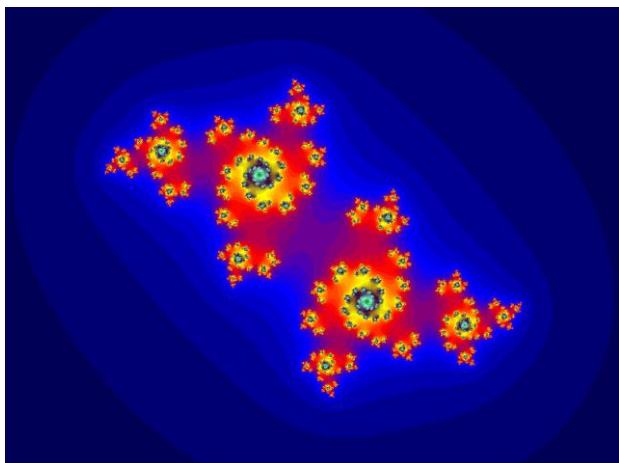


Figure 3. The Julia Set for function $Z_{n+1} = Z_n^2 - 0.7 - 0.3i$, maximum iterations 256, in the area $(-2, 2) * (-1.5i, 1.5i)$

The J-Set is also a fractal except in case a is equal to -2 or 0 . It has not been shown that these are the only exceptional values.

We should notice the difference in the definition of these two types of sets. When calculating the M-Set the initial value of P_c^n for each point of the complex plain is 0 , opposite to this when calculating the J-Set the initial value of Z_0 is equal to its position in the complex plain. The second difference is the way the constants c and a are used in the definition of the sets of quadric polynomials. For the J-Set a has a constant value that is used for the calculation of the entire set. In the case of the M-Set the value of c , has a different value for each tested point, and is equal to the corresponding point of the complex plain.

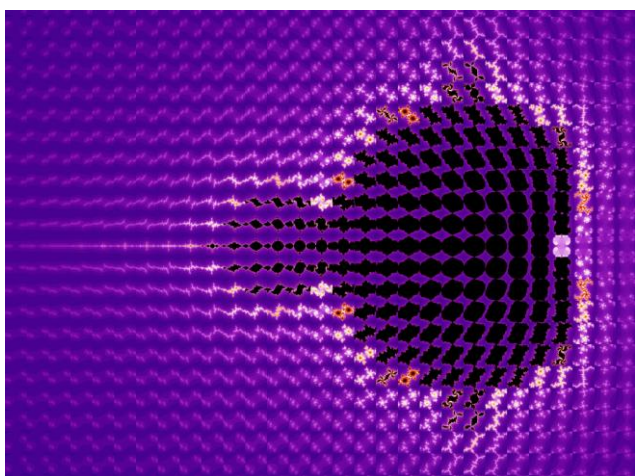


Figure 4. Map of Julia Sets for function $Z_{n+1} = Z_n^2 + c$

The close relation between these sets is evident when we observe the mapping of J-Sets according to the constant a to the complex plain in Fig. 4. The mapping of J-Sets closely resembles the M-Set. This relationship has also inspired the definition of a four dimensional M-Set [17], [18]. In this definition z and t coordinates are added as the initial value of Z_0 ($z = \text{real}(Z_0)$, $t = \text{imag}(Z_0)$), and the same formula is used as for calculating the M-Set with an initial value instead of 0 . The M-Set is the intersection of the four dimensional set with the plain $\alpha(z=0, t=0)$. J-Sets can be defined as intersections with planes $\beta_a(x = \text{real}(a), y = \text{imag}(a))$.

3 Buddhabrot Display Algorithm

The original M-Set is defined with $Z_{n+1} = Z_n^2 + c$, but the concept has been extended to an arbitrary function F and a corresponding M-Set generated with the iterative method $Z_{n+1} = F(Z_n, c)$. We display the M-Set in a given area $A = (xmin, xmax) * (ymin, ymax)$ of the complex plain and a given resolution h which defines a grid $G(A, h)$. The standard algorithm for displaying the M-Set in G is given in the following pseudo code

```
foreach (Point ∈ G){
  Iteration = 0;
  Z = Point
  while ( (|Z|^2 < bound)
    and (iteration < MaxIteration)){
    Z = F(Z, Point)
    Iteration++
  }
  Color(Point) = ColorIndex(iteration)
}
```

This is the simplest algorithm for generating a representation of the Mandelbrot set and is known as the "escape time" algorithm. With this display algorithm we show not only if a point is a member of the M-Set but also for non-member points the number of iterations they have been bounded (Fig. 2). The number of iterations gives us the information how quickly $P_c^n(z)$ grows for point c .

In the basic coloring algorithm we only use the number of iterations that $|Z|$ was bounded, but we do not use the information of which points Z have been passed and how many times, during these steps. The Buddhabrot method extends basic display algorithm by preserving this data.

The idea is adding a two-dimensional array corresponding to grid G , and counting the number of "hits" Z has made on elements of the grid during its

iterations. The Buddhabrot algorithm is shown in the following pseudo code.

```

Counter.SetToZero();
for (MaxNumberOfPoints){
    Point = Random element of area A
    Iteration = 0
    Z = Point
    Steps.Empty()
    while ( ( $|Z|^2 < bound$ )
        and (iteration < MaxIteration)){
        Steps.Add(Z)
        Z = F(Z, Point)
        Iteration++
    }
    if((iteration=MaxIteration)=OUTSIDE){
        foreach(Z ∈ Steps)
            Counter (Z) += 1
    }
}
foreach (Point ∈ G){
    Color(Point) = ColorIndex(Counter(Point))
}

```

We wish to point out some differences compared to the basic algorithm. First, instead of using the nodes of the grid, we use a fixed number of random points inside area *A*. This is important because if we took just nodes of a square grid, a possibility exists that they shall have some common properties and our “statistical” image will not be correct in that case. For each tested point we save the iteration steps in an array *Steps*. This is done because we create two separate images: one for points inside (Fig. 5), and one for points outside (Fig. 6) the M-Set.

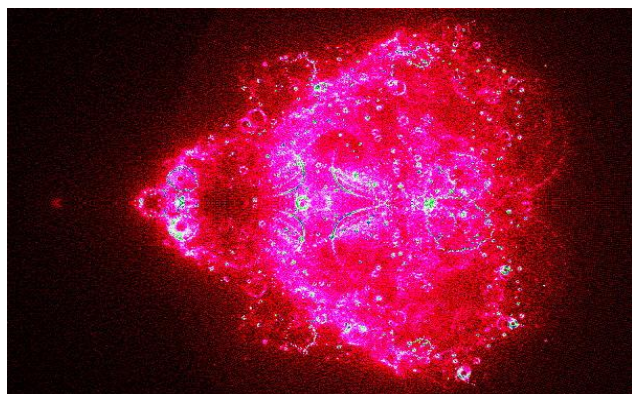


Figure 5. The Buddhabrot coloring, outside points, maximum iterations 25600, points rendered $24 \cdot 10^4$

Counter is a two-dimensional array that corresponds to blocks of grid *G*, it is used as the counter of “hits”. After finishing the calculations loop for a

point, depending on whether it is a member of the M-Set, we update the *Counter* with the points that have been crossed. After *MaxNumberOfPoints* has been reached we use the *Counter* for coloring the screen. The image acquired with this algorithm for points out of the M-Set is named Buddhabrot.

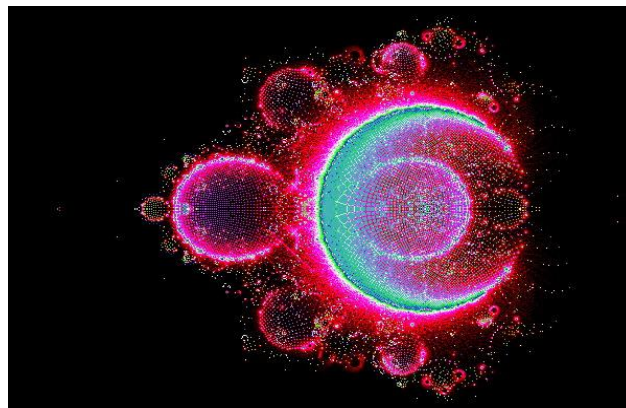


Figure 6. The Buddhabrot coloring, inside points, maximum iterations 25600, points rendered $6 \cdot 10^4$

4 Calculation-Path Display Algorithms

In this section, we show our novel concept for displaying the M-Set. We extend the idea of preserving information from the calculations of the fractal image. We count not only which points have been visited during iterations steps, but we also track the “calculation path”. The path from Z_n to Z_{n+1} is seen as a line connecting these two points. The calculation path of point *Z* is an array of lines connecting successive Z_i appearing in the iterative method for checking if the point belongs to the M-Set. We named this display method Calculation-Path. It is shown in the following pseudo code.

```

Counter.SetToZero();
foreach (Point ∈ InputGrid){
    Iteration = 0;
    Z = Point
    CalculationPath .Empty()
    while ( ( $|Z|^2 < bound$ )
        and (iteration < MaxIteration)){
        CalculationPath.Add(Z)
        Z = F(Z, Point)
        Iteration++
    }
    if((iteration = MaxIteration) = OUTSIDE){
        for (i=0; i<CalculationPath.length-1; i++){
            foreach( $P \in \overline{Z_i, Z_{i+1}} \cap G$ )
                Counter (P) += 1
        }
    }
}

```

```

    }
  }
}
foreach (Point ∈ G){
  Color(Point) = ColorIndex(Counter(Point))
}

```

In this algorithm we also use *Counter* for counting “hits”, and different images are created for images outside (Fig. 7) and inside (Fig. 8) of the M-Set.

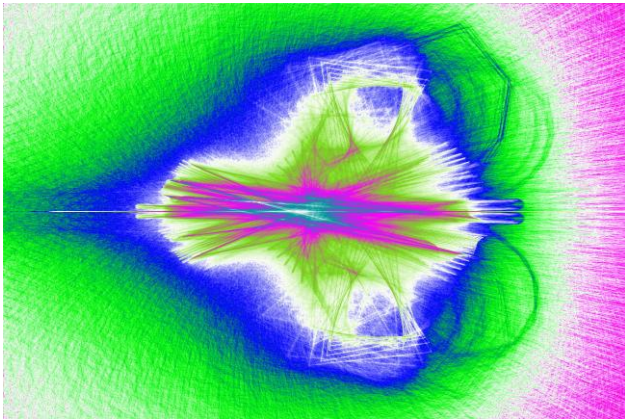


Figure 7. The Calculation-Path coloring, outside points, max iterations 256000, input grid 300*200, output grid 1200*800

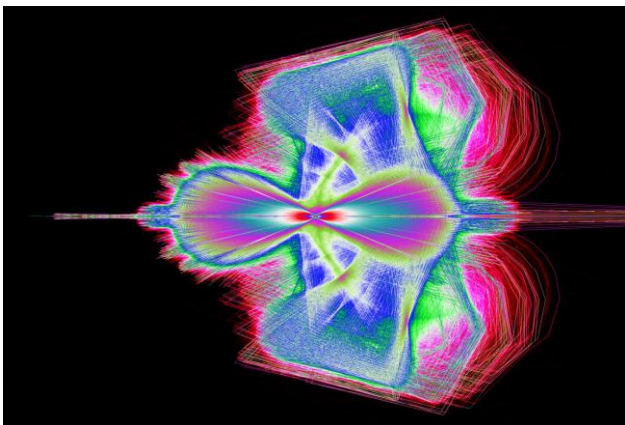


Figure 8. The Calculation-Path coloring, inside points, max iterations 256, input grid 300*200, output grid 3000*2000

The main difference is in the way the iteration step points are handled, instead of just incrementing *Counter* elements corresponding to these points we increment all the grid elements belonging on the line $Z_i Z_{i+1}$, excluding the point Z_i . We wish to point out that when implementing this algorithm, the speed of the line incrementing is of great influence to the overall calculation time. Due to the fact that line incrementing is the same as drawing lines on

Counter we used the Bresenham’s line algorithm [19] and Fast-Clipping algorithm [20] to optimize this process. The second big difference is the use of two grids, one for the input points, and a second one for the finalized image. Different combination of these grids gives different effects to the final image.

We also propose a variation of the Calculation-Path image algorithm, in which we also take into account the length of $Z_i Z_{i+1}$. We use the following method of incrementing

```

foreach(P ∈  $\overline{Z_i Z_{i+1}} \cap G$ )
  Counter (P) += Const / Length( $\overline{Z_i Z_{i+1}}$ )

```

This is a natural extension; it could be understood as the time spent at each point, but other functions can also be used instead of $Const/Length(\overline{Z_i Z_{i+1}})$. In our images we used $Const/Length(\overline{Z_i Z_{i+1}})^2$ to even more emphasize the shorter paths. We call images created with this variation Time-Spent (Fig. 9, 10).

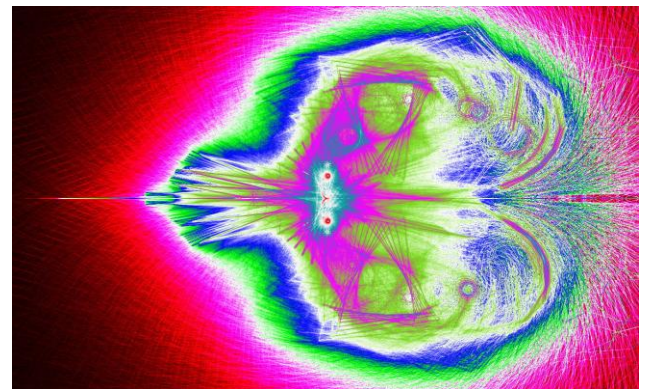


Figure 9. The Time-Spent coloring, outside points, max iterations 256000, input grid 300*200, output grid 1200*800

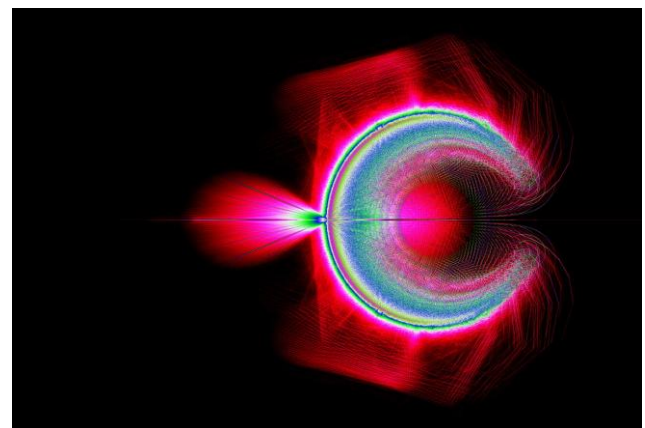


Figure 10. The Time-Spent coloring, inside points, max iterations 256, input grid 300*200, output grid 3000*2000

5 Calculation-Path Fractal Images

Due to the similarity of algorithms for generating the BUD, CP and TS images we shall compare some of their properties. We first notice that for CP and TS images we need a lower number of points for creating images due to the fact that we are drawing lines instead of points. When generating images with a large number of points, images can even lose details due to intensive overlapping of paths. In this case, more statistical information is presented, but individual paths are less visible. When creating the new type images, instead of using random points we used points of a grid. This approach gave a more representative sampling of the space when a small number of points were tested. Images acquired when sampling the space with a very sparse grid are very interesting because they visualize the movement of individual points and different behavior in different regions (Fig. 11, 12).

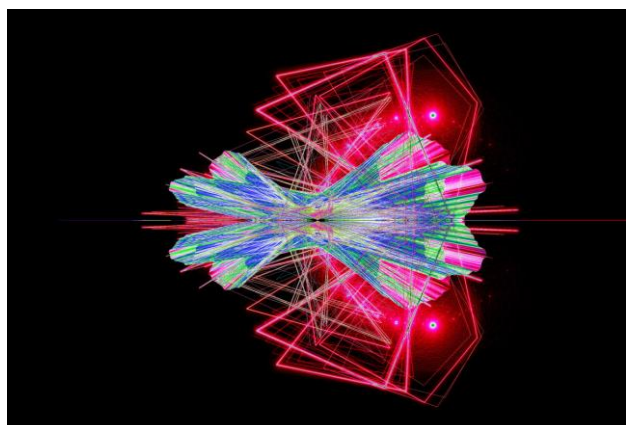


Figure 11. The Calculation-Path coloring, inside points, max iterations 25600, input grid 60*50, output grid 3000*2000

In the case of tests with a large number of points, a random selection is better for the same reasons as for the Buddhabrot method.

Alex Boswell's method for vastly increasing the speed of rendering of highly zoomed regions [21] using the Metropolis-Hastings algorithm [22] is less productive in our case because distribution of paths is more complicated than the distribution of visited points. Opposite to the Buddhabrot images where zooming into them without this optimization resulted in an extremely big increase in calculation time, for images created with these technique it can be done in approximately the same time due to the use of lines instead of points. The zoomed image sometimes slightly differ in color from the reign selected in the start image because in the new output grid line pairs that have intersected at the same point now might intersect at points besides each other.

Using this possibility we can create large scale images without a large increase in calculation time because it depends mostly on the size of the input grid.

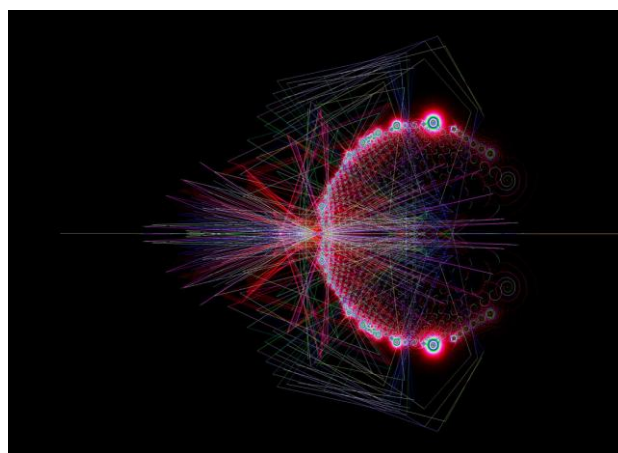


Figure 12. The Time-Spent coloring, inside points, max iterations 25600, input grid 60*50, output grid 3000*2000

When using this method of displaying the M-Set we can observe some new properties like connections between different parts of the set. This display method can be used on Julia sets also.

6 Fractal Generator Program

We created fractal generator software for creating Buddhabrot(BUD), Calculation-Path(CP) and Time-Spent(TS) fractal images. It also has the possibility of creating standard Mandelbrot and Julia type images. An alpha version of this software and source code can be downloaded from

<http://mail.phy.bg.ac.yu/~rakaj/home/>. The software has been developed in C# using Microsoft Visual Studio 2005. The software is designed as a plug-in system. It consists of the main program and plug-ins for new formulas.

To manipulate these type of images we developed a graphical user interface (GUI) that consists of parts that control different properties of image creation. In Fig. 13 on the left side we can see an image created with our software. This part of the GUI is also used for selecting areas that shall be rendered. This selection is done in two directions. First, we can select a part of a M-Set from which we wish to create BUD, CP or TS images for points inside or outside of the set. Second, we can also select the area of the BUD, CP or TS images that we wish to be calculated. Using this feature, we can also zoom in to images.

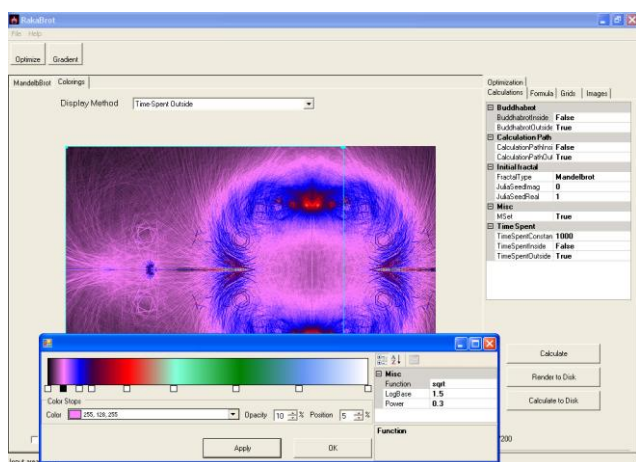


Figure 13. Screenshot of developed fractal generator Software

The second part of the GUI gives us further control over more properties of the calculation algorithm. These properties are divided into several tabs. In the first tab, named “Calculations”, we set which types of display methods will be calculated in the iterative algorithm for creating images. This gives great decries in calculation time when more than one image is created, because the M-Set and the calculation path is calculated only once, and this information can be used for several images. In this section we also have control if the Mandelbrot or Julia type fractal will be used.

The second tab gives us the control over the formula properties (Fig. 14). As previously mentioned, different types of formulas are held in different plug-ins so this part of the GUI also controls there selection and activation of them.

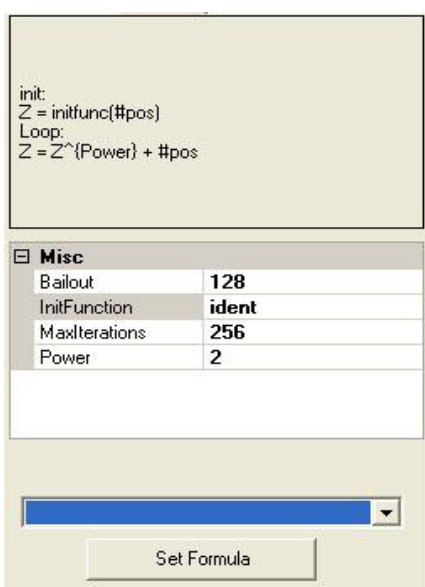


Figure 14. Screenshot of GUI tab for formula manipulation.

This part of the GUI gives us basic information about the plug-in (top part of Fig. 14) and the control over its parameters. It gives us the method for calculating the initial value of z , and the formula that will be used in the iteration loop. All possible plug-ins have the option of setting the maximal number of iterations and bound for $|Z_n|$ in the parameter control section. Depending on the type of plug-in, we have other parameters that could be integer, real or function type. Function type parameters are parameters that can have the value of basic functions like: identity, sinus, cosine, exponential... Using these parameters gives a wide range of different formulas that could be calculated by the same plug-in.

The next tab “Grids” is used in combination with the left side of the GUI. As said before, the selection is defined with an input area, or in other words the section of the complex plain in which we shall be searching for point inside the M-Set. The output area is equivalent to the part of the complex plain in which we shall be counting the hits for new images. In this tab we can enter more precisely the area directly setting numerical values. The second function of this tab is to fully define these grids by setting the number of grid points on the x and y axis. By defining the grid, we also define the dimensions of the output images.

Buddhabrot, Calculation-path and Time-Spent display method all need a large amount of computation time for creating images. In addition, often the acquired images do not have the correct coloring for us to observe the properties of the M-Set. To avoid calculating the images again through the iterative method for the M-Set, we added to our fractal generator a possibility of preserving the intermediate numerical image. The numerical image is a real value of each point in the grid. For the manipulation of this type of data we added an extra tab “Images”. Using this feature multiple properties of calculated M-Sets could be observed with minimal extra execution time.

In our software, we added a technique that greatly reduces the calculation time for calculating these images. In most of the cases, we need only the inside or outside images for some M-Set. A great part of our calculation time is used to calculate points that in the end will not be used for generating the final image because they are outside/inside of the M-Set if we are calculating the inside/outside image. The overall calculation could be greatly decreased if these point could be quickly recognized or even excluded from calculation. One of the properties of the $P_c^n(z)$ is that it is value can change significantly for very small changes of c , but this is only true near

the border of the M-Set. In areas not near the border the behavior is stable. We used this property of the M-Set to greatly optimize the calculation of BUD, CP and TS large images, by developing an algorithm that needs some assistance from the user.

The optimization has several steps. First, the user calculates the image in some low resolution. Using this image the user can select areas of the M-Set that will be inside or outside of it depending on which type of images will be calculated. The selection consists of an array of rectangles with an extra property that specifies if a point should be inside or outside of it; an example of this selection can be seen in Fig. 15.

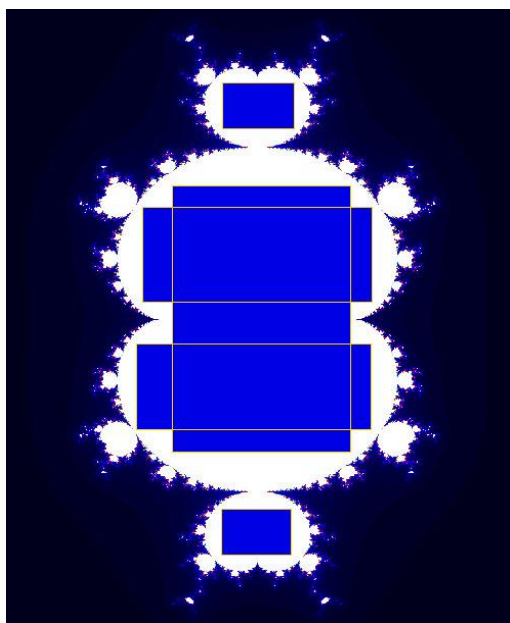


Figure 15. An example of selection of areas that will not be used for calculation of outside BUD, CP and TS images for M-Set corresponding to $Z_{n+1} = Z_n^3 + c$. White points are points inside the set. Blue rectangles with yellow border are the selection.

The creation of this selection is done using our GUI. We use the “Optimization” tab in which we manipulate the array of selected rectangles. Individual rectangles are adjusted directly setting numerical values for top left point and the height and width of the rectangle in this tab. These values can also be set by using the right side of the GUI selecting areas from the image. The algorithm that optimizes the calculation of these images is given by the following pseudo code.

```
Pos.Real = Start.Real
for (j = 0; j < Grid.Height; j++){
  Pos.Complex = StartComplex;
  for (i = 0; i < Grid.Width; i++){
```

```
    if ((Pos ∈ Selection){
      Pos ∈ rect
      Calculate number of steps(grid points)
      from Pos to right end of rectangle(rect)
      i += steps-1;
      mPos.Real += Step.Real*steps;
    }else{
      Z = mPos;
      Init(Z);
      Loop(Z);
      mChangeImages(i, j);
      mPos.Real += Step.Real;
    }
  }
  mPos.Complex += Step.Complex;
}
```

The change to the basic algorithm is in adding extra criteria for selecting which points will be used from the grid. This criteria is combined with an improved method for moving through the grid. As in the basic algorithm we have a nested loops for moving through X and Y axis for testing all the points in the grid, but we also have a test to see if a grid point belongs to any of the selected rectangles. If point $A(x,y)$ belongs to rectangle R with left point at position p and width w , the testing point will be advanced to position $(p+w,y)$. In the future, we plan to implement similar fully automated algorithm. These selections rectangles can, besides for optimization, be used for creating images with parts of the M-Set excluded from the calculation.

To improve the analyses of different properties of CP and TS images we have implemented significant coloring control to our GUI. This is done in two directions, first with easy control of palette creation in a similar fashion to other image editing and fractal generator software like Adobe Photoshop, Ultra Fractal and Apophysis. The second control is adding methods for the conversion of the values inside the numeric image into palette indexes. We have implemented the use of following functions x , \sqrt{x} , $\text{pow}(x, \text{base})$, $\log(x)$, $\tan(x)$. We have created this control extending an open source component available at the code project web-site [23].

In our software we also implement features that can be used to better understand the 4-dimensional M-Set. We do this by making it possible to create mappings, or in other words a visual representation of intersections of 2-dimensional planes with the 4-dimensional M-Set. A mapping shows us the change that occurs when these planes are chosen parallel to two of the axis x , y , z and t , or in other words, that

they can be defined with the following way α (axis1= a , axis2= b). When creating a mapping parameters a and b are selected on some uniform grid for areas (A_1, A_2) and (B_1, B_2) . This is better understood if we observe Fig. 16, 17. In both of these figures the center of the image corresponds to the intersection with the plain that holds coordinate center $(0 + 0*i)$ of the complex plane.

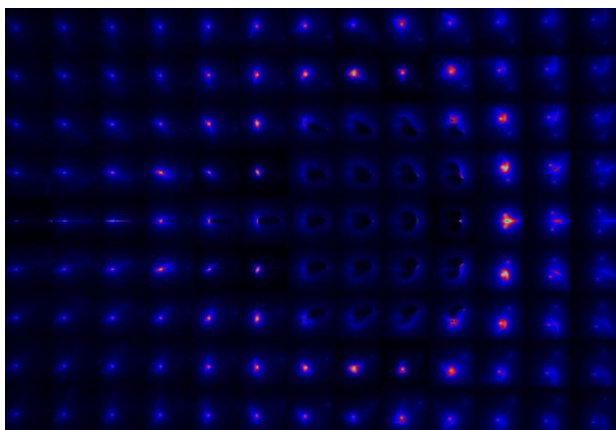


Figure 16. Mapping of intersecting planes corresponding to J-Sets, β ($x=a, y=b$), where $-2 < a < 1$ and $-1 < b < 1$. Using the Time-Spent display method

The most interesting images are acquired when we use planes parallel to x and y axis which corresponds to different J-Sets or to z and t which corresponds to M-Sets with different initial values. In our software, it is possible to create mappings for any other combination of axis. It is possible to create mappings for all the previously mentioned methods for displaying M and J sets.

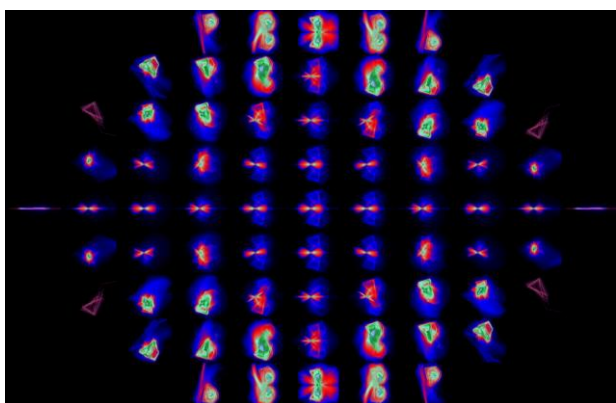


Figure 17. Mapping of intersecting planes corresponding to M-Sets with different initial value, α ($z=a, t=b$), where $-1.75 < a < 1.75$ and $-1 < b < 1$. Using the Time-Spent display method

7 Conclusion

In this paper we have presented an algorithm for creating images that make it possible to observe new aspects of the M-Set. These images display different information about the M-Set than previously developed display methods like Buddhabrot and Pickover Stalks did. The Calculation-Path images show as the connection between different areas of the M-Set, and give us statistical information about the iterative process for calculating the members of the M-Set. This is done by preserving previously ignored information of the order of points appearing in the steps of iterative algorithm. We created two variations of images. The first one only uses the directions and frequency of calculation paths. The second variation also takes into account the length of these paths. This display method can also be used on Julia type fractals.

We have developed a fractal generator program that enables the creation and thorough analysis of images created using Buddhabrot, Calculation-Path and Time-Spent display techniques. Our software is a plug-in system for which we have created several modules which explore a wide range of different types of formulas. The execution time needed for creating these images is great due to the nature of the algorithms. In our software we have implemented several optimizations that make creation of large scale images possible. We have also incorporated features that make possible to analyze the 4 dimensional concept for the M-Set.

In the future we wish to adopt previously developed algorithms for the Buddhabrot method like Alex Boswell optimization method and the 4D Buddhabrot Hologram to the Calculation-Path concept. We also wish to analyze the connections of images created with these new techniques for pairs of M-Set and corresponding J-Sets. Due to the similarity of the algorithm to Pickover Stalks which proved its value in biology through biomorphs, we believe research in this direction is justified.

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