#### **Regular** Article

# Positron transport in $CF_4$ and $N_2/CF_4$ mixtures<sup>\*</sup>

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**Abstract.** In this paper we present our recently compiled set of cross sections for positron scattering in CF<sub>4</sub>. Using this set of cross sections as an input in our multi term Boltzmann equation analysis of positron transport in CF<sub>4</sub> and N<sub>2</sub>/CF<sub>4</sub> mixtures, we calculate various transport properties as a function of the reduced electric field  $E/n_0$ . Values and general trends of the mean energy, drift velocity and diffusion coefficients as a function of  $E/n_0$  are reported here. Emphasis is placed on the explicit and implicit effects of positronium (Ps) formation on the drift velocity and diffusion coefficients. Two important phenomena arise. Firstly, for certain regions of  $E/n_0$  the bulk and the flux components of the drift velocity and longitudinal diffusion coefficient are markedly different, both qualitatively and quantitatively. Secondly, and contrary to previous studies in positron swarm studies, there is negative differential conductivity (NDC) in both the bulk and flux drift velocity. The variation of the drift velocity with  $E/n_0$  and origin of the NDC are addressed using physical arguments. Cross sections and transport data for positrons in CF<sub>4</sub> and N<sub>2</sub>/CF<sub>4</sub> mixtures presented in this work are compiled and evaluated with an aim of improving the efficiency of positron traps.

### 1 Introduction

Positron physics is a growing area of research [1-3], ranging from medical science [4] and astrophysics [5] to material science [6]. The fundamental data on positron behavior in gaseous, liquid and solid matter under different conditions and in various environments are rapidly increasing and new possibilities for applications are emerging (positherapy [7,8] being an exotic example). Modeling of these applications is necessary because it can provide a better insight into the fundamental phenomena, and enable optimization of such applications. Modeling, however, usually requires two types of input data. The first are complete sets of cross sections for collisions of positrons with individual atoms and/or molecules of matter in question. The second are transport data, relying on scattering data, that are the source of information about a group or ensemble properties of positrons (swarm) traveling through the medium, such as their mean energy, drift velocity and diffusion. Unfortunately, the experimental data on transport coefficients for positrons [9] are very limited in the

literature and only a few groups in the world are directly involved in modeling of positron transport. A review of the history and current status of positron swarm experiments have been recently outlined by Charlton [9] and Petrović et al. [10].

The invention of the Surko trap [11,12] has made a revolutionary breakthrough in measuring positron cross section data and provided a renewed momentum to positron physics. Before such traps, it was impossible to achieve positron beams with sufficient flux and energy resolution. Nowadays Surko traps are the main source of positrons for positron scattering experiments from atoms and molecules [13–17] and other experiments [18–21]. Those range from the creation of positronium (Ps) molecules [22] to the formation of anti-hydrogen atoms [22–24], and even experiments aiming to achieve Ps Bose-Einstein condensates [25].

The positron scattering data resulting from experiments based on the Surko trap together with the theoretical cross-section calculations have triggered a series of swarm oriented studies where calculation of transport parameters for positrons in various gases under the influence of both the electric and magnetic fields plays the central role [26–31]. Positron beams in the Surko trap evolve quickly into a 'swarm-like' particle distribution [32], so the concepts and theory of charged particle swarms are highly appropriate to model the behavior of positrons in

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this trap. Although the existing design of the trap currently facilitates many different kinds of measurements, it is always desirable to try to store more positrons. Modeling of the positron trap operation [32] is important in order to better understand the underlying mechanisms and improve the trapping efficiency, thermalization time and beam radius. One of the illustrative outcomes of recent simulations was a suggestion that a trap setup with  $CF_4$  as the only trapping gas at incoming energies far below the threshold for Ps formation, will achieve higher efficiency [33].

The most efficient buffer gas in positron traps so far was N<sub>2</sub>, with the addition of CF<sub>4</sub> in the final stage of the trap. The threshold for excitation of the  $a^1\Pi$  electronic level of N<sub>2</sub> in positron impact is conveniently positioned just below the threshold for Ps formation [34]. Cross sections for these two processes are of similar magnitude, and therefore a huge percentage of positrons hit the electronic excitation, lose their energy and never get a chance to form Ps and disappear from the swarm [28]. Cooled positrons are transferred into the final stage of the trap where they collide with CF<sub>4</sub> molecules and lose significant amount of their energy through the vibrational excitation of CF<sub>4</sub> [13].

In order to calculate positron transport coefficients either by Monte Carlo simulation [35,36] or through a numerical solution of Boltzmann's equation [37,38], it is necessary to have a complete set of cross sections for positron scattering. We regard a set of cross sections which covers all important processes and provides a good balance of number of particles, momentum and energy within the swarm to be complete. In the case of electron swarms there are recommended sets for many atoms and molecules which are being revised and updated regularly, as illustrated for example in reference [39], but for positrons such data are often missing in the literature [40]. In those cases it is unavoidable sometimes to make educated guesses, for example to use available data for positron scattering with similar atoms/molecules or even data for electron scattering. The basic differences between electron and positron scattering can be summarized in the following points [1]: (i) the short-range positron-atom interaction is repulsive in contrast to the attractive electron-atom interaction: (ii) for positrons the exchange interaction does not exist: and (iii) positronium (Ps) formation channel is very strong and unique for positron interaction. Basically, the most significant differences between the positron cross sections and electron cross sections are in the low-energy range, and that is exactly the range that we are interested for in this paper. Therefore, in creating cross section sets for positrons in  $N_2$  and  $CF_4$ , we have been forced [28,40] to carefully compile the best available data from the literature and to find the best ways to overcome the problems arising from the fundamental data that are not available.

In this paper we present our set of cross sections for positron scattering in  $CF_4$ , with the primary aim for use in modeling of cooling processes of positron cloud in the buffer gas positron accumulators. Using this set and previously developed and published set of cross sections for



Fig. 1. Cross sections for positron scattering in  $CF_4$ .

positron scattering in N<sub>2</sub> [28,40], we have calculated transport properties for positrons in pure CF<sub>4</sub> and in mixtures of N<sub>2</sub> with different percentage contribution of CF<sub>4</sub>. One of the most important motivational factors for this work was the development of a database of collisional and transport data for the recently suggested pure CF<sub>4</sub> trap [33]. We believe that data presented in this paper can additionally support the ideas presented in reference [33]. Transport properties for positrons have been calculated using a multi term Boltzmann equation analysis [36–38]. A Monte Carlo simulation technique [31,36] is used as an additional method to test some of the interesting and atypical phenomena observed in the positron transport in pure CF<sub>4</sub> and CF<sub>4</sub>/N<sub>2</sub> mixtures.

#### 2 Cross sections for positrons in CF<sub>4</sub>

The cross section set for positron interactions with  $CF_4$ is shown in Figure 1. This set includes cross sections for elastic collisions, Ps formation and inelastic processes (direct ionization, vibrational and electronic excitations). In creating this cross section set we primarily had in mind the important role that  $CF_4$  has as a cooling gas in Surkotype positron traps [11,12]. Therefore, this 'complete set' does not include the cross section for direct annihilation since it is not relevant for the behavior of positrons in the trap [32]. Currently,  $CF_4$  is used in the last (low energy) stage of thermalization in the Surko traps, and the approximate estimates of Ps formation, direct ionization and electronic excitation cross sections were not critical factors in the models. However, as there were suggestions that  $CF_4$  may be used as the sole trapping gas [33], and also for rotating wall applications [21,41], we have attempted to make the set as complete as is presently possible. We hope that this will also give motivation for experimentalists and theoreticians to provide the necessary data for the missing processes.

The total cross section is taken from the experiment of Makochekanwa et al. [17] and it covers the energy range from 0.2 to 1000 eV. It is extrapolated to zero energy using the theory of Nishimura and Gianturco [42]. The elastic cross section is obtained by subtracting all inelastic cross sections and the cross section for Ps formation from the total cross section.

To our knowledge, there is no data on the Ps formation cross section for  $CF_4$  in the literature. Therefore we have used the cross section for Ps formation in Ar (as suggested in Ref. [14]) shifted by the difference between the thresholds for this process in Ar and  $CF_4$ . The justification for this choice lies with the fact that cross sections for ionization are very similar in both shape and magnitude for Ar and  $CF_4$ . There is a new measurement of Ps formation cross section in Ar [15], which agrees in shape with [14] and slightly differs in magnitude (it is 9% higher at the peak). We believe that such a small difference will not greatly affect the positron transport data. Thresholds for Ps formation in Ar and  $CF_4$  are 8.955 eV [14] and 9.45 eV [17], respectively, so we added  $\Delta = 0.495$  eV to the energy scale of the Ps cross section for Ar. The other choice would have been the Ps formation cross section for  $CH_4$ , but to the best of our knowledge there is no Ps formation data for this molecule in the literature.

A similar situation exists for the cross section for direct ionization of  $CF_4$  via positron impact. Our starting point in constructing this cross section was the cross section for total ionization of  $CF_4$  (i.e. including all ionisation processes – direct and Ps-formation ionizations of the molecule) measured by Moxom et al. [43] in the energy range 6-47 eV. It was extrapolated to higher energies using the total ionization cross section for positrons in  $CH_4$ measured by Bluhme [44], shifted by the difference between the thresholds for this process in  $CF_4$  and  $CH_4$ . When shifted by this difference ( $\Delta = 3.303 \text{ eV}$ ), the CH<sub>4</sub> cross section smoothly merges with the  $CF_4$  cross section. We then subtracted the cross section for Ps formation previously described from the obtained total ionization cross section to get the direct ionization cross section for  $CF_4$  via positron impact. The threshold for this process is 16.25 eV. Since the most important role in positron trapping is played by the huge cross sections for vibrational excitations of  $CF_4$  in both the Surko trap and the recently proposed  $CF_4$  trap [33], we believe that approximate estimates of the Ps formation cross section and the cross section for direct ionization are not critical factors [13] and are sufficient for our modelling considered here.

Electronic excitations of the  $CF_4$  molecule induced by positron impact also play an important role in enhancing the percentage of trapped particles when  $CF_4$  is one of primary cooling gases [33]. Unfortunately, again there is a lack of data in the literature. One way to include electronic excitations into the cross section set is to take the effective cross sections for electronic dissociative excitations by electrons [45] and then to divide it by the factor of 4 [46] as positrons may only excite singlet transitions.

The most important process in the trapping of positrons in Surko trap [21,41] and in CF<sub>4</sub> trap [33] are

vibrational excitations of CF<sub>4</sub>. This cross section set includes cross sections for three of the four vibrational excitations [34]: v1 ( $E_{exc} = 0.113 \text{ eV}$ ), v3 ( $E_{exc} = 0.159 \text{ eV}$ ) and v4 ( $E_{exc} = 0.078$  eV). The cross section for vibrational excitation v2 ( $E_{exc} = 0.054 \text{ eV}$ ) is omitted because neither experimental nor theoretical data exist for this transition, for neither positrons and electrons presumably due to its small magnitude or as it was included in one of the other three processes. The energy range relevant for this process is still apparently out of reach for modern experiments [34]. The cross section for v3 has been measured in San Diego in the 0.1-2 eV energy range [34]. The cross section for v3 excitation of  $CF_4$  molecule by electron impact has also been measured in the same energy range and using the same apparatus. This experiment revealed that vibrational cross sections for electrons and positrons in  $CF_4$  are similar both in shape and magnitude, and this is confirmed by calculations based on the Born approximation [34]. Consequently, we have extrapolated the cross section for the v3 vibrational mode using results for electrons obtained by a swarm method [47]. Since the positron data for the vibrational modes v1 and v4 are missing, we have used the available data for electrons [47].

According to the symmetry,  $CF_4$  belongs to the spherical top molecules and also to cubic point group  $T_d$ . It therefore has no net dipole moment. A pure rotational spectrum cannot be observed by absorption or emission spectroscopy because there is neither permanent dipole nor quadrupole moment [48] and thus rotation may not be induced by the electromagnetic field of an incident photon. Further, the polarizability is isotropic so that a pure rotational transition cannot be observed by Raman spectroscopy [49]. Thus in all analyses for electron swarms in gaseous  $CF_4$  and for plasma models the rotational interaction has been completely neglected [47, 50, 51]. Led by this we have used the same approximation for positrons, i.e. we have neglected rotational excitation. One should however bear in mind that in theoretical work of Varella et al. [52] cross sections have been calculated for rotational excitation of several transitions for electron  $CF_4$  collision, with large changes in rotational quantum number. They used an ab initio Schwinger multichannel method in conjunction with norm-conserving pseudo-potentials. Their calculations give relatively high cross sections ( $\sim 10^{-16} \text{ cm}^2$ ) for rotational excitations at high energies where the pure rotational energy transfer is estimated to be  $\sim 10^{-5}$  eV per collision. This number is very small comparing to vibrational energy transfer. In principle electrons could excite optically forbidden transitions so the magnitude and the role of rotational excitation in highly symmetric molecules is still an open issue. Apart from the results of Varella et al. [52] to our knowledge there are no available rotationally resolved cross sections in the literature for  $CF_4$ . Therefore we did not try to take into account this interaction channel separately. In our set of cross sections rotational excitations of CF<sub>4</sub> by positron impact, if present, are implicitly included through elastic cross section through their inclusion in measured total cross section.

Another comment related to this issue should be made here. In case of electron transport in plasmas the mean energies are such that energy balance is dominated by vibrational excitation and the high energy tail of the velocity distribution function is determined largely by the electronic excitation. Thus taking or failing to take the rotational contribution is of little consequence. In the case of positrons, however, we are interested in thermalization and thus we need a good energy balance in the approach to the thermal energy where rotational excitation is the dominant inelastic process.

# **3** Theoretical methods

All information on the drift and diffusion of positrons in neutral gases is contained in the positron phase-space distribution function  $f(\mathbf{r}, \mathbf{c}, t)$ , where  $\mathbf{r}$  represents the spatial coordinate of a positron at time t, and  $\mathbf{c}$  denotes its velocity. The distribution function  $f(\mathbf{r}, \mathbf{c}, t)$  is evaluated by solving Boltzmann's equation:

$$\left[\partial_t + \boldsymbol{c} \cdot \nabla \boldsymbol{r} + \frac{e}{m} \boldsymbol{E} \cdot \nabla \boldsymbol{c}\right] f\left(\boldsymbol{r}, \boldsymbol{c}, t\right) = -J\left(f, f_0\right), \quad (1)$$

where  $\partial_t$ ,  $\nabla_r$  and  $\nabla_c$  are the gradients with respect to time, space and velocity, while e and m are the charge and mass of the positron and E is the magnitude of the electric field. The right-hand side of equation (1)  $J(f, f_0)$ denotes the linear positron-neutral molecule collision operator, accounting for elastic, inelastic and nonconservative (e.g. Ps formation and/or annihilation) collisions. For elastic collisions we use the original Boltzmann collision operator [53], while for inelastic collisions we prefer the semiclassical generalization of Wang-Chang et al. [54]. The collision operators for non-conservative collisions are discussed in [55,56].

The solution of equation (1) is found by expanding  $f(\mathbf{r}, \mathbf{c}, t)$  as sums of products with the directional dependence of  $\mathbf{c}$  contained in spherical harmonics  $Y_m^{[l]}(\hat{\mathbf{c}})$ , the spatial distribution contained in  $G_{\mu}^{(s\lambda)}$ , the sth application of the spatial gradient operator on  $n(\mathbf{r}, t)$ , and the speed distribution contained in an expansion discussed below. Thus, we have:

$$f(\boldsymbol{r},\boldsymbol{c},t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{s=0}^{\infty} \sum_{\lambda=-s}^{s} f(lm|s\lambda;c) Y_{m}^{[l]}(\hat{\boldsymbol{c}}) G_{m}^{(s\lambda)}.$$
(2)

The coefficients  $f(lm|s\lambda; c)$  are functions of the speed c, obtained by the expansion

$$f(lm|s\lambda;c) = \omega(\alpha,c) \sum_{\nu'=0}^{\infty} F(\nu lm|s\mu;\alpha) R_{\nu'l}(\alpha,c), \quad (3)$$

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where

$$\omega(\alpha, c) = \left(\frac{\alpha^2}{2\pi}\right)^{3/2} \exp\left\{\frac{-\alpha^2 c^2}{2}\right\},\tag{4}$$

$$\alpha^2 = \frac{m}{kT_b},\tag{5}$$

$$R_{\nu l}(\alpha c) = N_{\nu l} \left(\frac{\alpha c}{\sqrt{2}}\right)^l S_{l+1/2}^{(\nu)} \left(\frac{\alpha^2 c^2}{2}\right), \qquad (6)$$

$$N_{\nu l}^{2} = \frac{2\pi^{2/3}\nu!}{\Gamma(\nu + l + 3/2)},$$
(7)

and  $S_{l+1/2}^{(\nu)}(\alpha^2 c^2/2)$  are Sonine polynomials.

Using the orthonormality conditions of the spherical harmonics and modified Sonine polynomials, the following hierarchy of kinetic equations follows:

$$\sum_{\nu'=0}^{\infty} \sum_{l'=0}^{\infty} \left[ \partial_t \delta_{\nu\nu'} \delta_{ll'} + n_0 J_{\nu\nu'}^l(\alpha) \delta_{ll'} - R_a \delta_{\nu\nu'} \delta_{ll'} + ia\alpha (l'm10|lm) \langle \nu l| |K^{[1]}| |\nu'l' \rangle - n_0 J_{0\nu'}^0(\alpha) F(\nu l0|00; \alpha) \times (1 - \delta_{s0} \delta_{\lambda 0}) \delta_{l'0} \delta_{m0} \right] F(\nu'lm|s\lambda; \alpha) = X(\nu lm|s\lambda; \alpha), \qquad (8)$$

where  $R_a$  is the rate for Ps formation. The reduced matrix elements of the collision operator  $J^l_{\nu\nu'}(\alpha)$  and velocity derivative  $\langle \nu l | | K^{[1]} | | \nu' l' \rangle$  are defined by equations (11) and (12) given in reference [56] while (l'm10|lm) is a Clebsch-Gordan coefficient. The explicit expression for the RHS are given in references [37,56].

The hierarchy of kinetic equations (8) is solved by truncation of the  $\nu$  and l indices to  $\nu_{\text{max}}$  and  $l_{\text{max}}$ , respectively. These values are independently increased until the desired convergence is obtained.  $T_b$  is not equal to the neutral gas temperature (the two-temperature method) and is used as a free parameter to optimize the convergence. After truncation, we have a hierarchy of coupled complex equations. This sparse system of equations is solved using standard sparse inversion routines.

The "two term" approximation is based upon the choice of setting the upper bound on the summation in (2) to  $l_{max} = 1$  and formed the basis for the classical theory of light charged particle transport properties. It is entirely inappropriate when the distribution function deviates substantially from isotropy in velocity space, and this is exactly what may happen for positrons in CF<sub>4</sub>. In Section 4.2.4 we study the convergence trend for various transport coefficients as a function of  $E/n_0$  for positrons in CF<sub>4</sub>.

#### 3.1 Transport properties

The transport coefficients of interest (rate coefficient for Ps formation  $k_{Ps}$ ; bulk drift velocity W; longitudinal  $(D_L)$  and transverse  $(D_T)$  diffusion coefficients) are related to

the calculated moments via

$$k_{Ps} = n_0 \sum_{\nu'=0}^{\infty} J^0_{0\nu'}(\alpha) F(\nu'00|00), \qquad (9)$$

$$W = \frac{i}{\alpha} F(010|00) - in_0 \sum_{\nu'=1}^{\infty} J^0_{0\nu'}(\alpha) F(\nu'00|11), \quad (10)$$

$$D_{L} = -\frac{1}{\alpha} F(010|11) - n_{0} \sum_{\nu'=0}^{\infty} J_{0\nu'}^{0} \left[ F(\nu'00|20) - \sqrt{2}F(\nu'00|22) \right], \quad (11) D_{T} = -\frac{1}{\alpha} F(011|11) - n_{0} \sum_{\nu'=0}^{\infty} J_{0\nu'}^{0} \left[ F(\nu'00|20) + \frac{1}{\sqrt{2}}F(\nu'00|22) \right],$$

where the components involving summations constitute the explicit effects of Ps formation on the transport coefficients, while the remainders constitute the flux contribution.

The spatially homogeneous mean energy  $\varepsilon$  and gradient energy vector  $\gamma$ , defined through a density gradient expansion of the averaged positron energy  $\epsilon$  ( $\mathbf{r}, t$ )

$$\epsilon \left( \boldsymbol{r}, t \right) = \frac{1}{n \left( \boldsymbol{r}, \boldsymbol{t} \right)} \int \frac{1}{2} m c^2 f\left( \boldsymbol{r}, \boldsymbol{c}, t \right) d\boldsymbol{c} = \varepsilon + \gamma \cdot \frac{\nabla n}{n} + \dots,$$
(13)

play a pivotal role in a qualitative understanding of the effects associated with the explicit influence of Ps formation on the positron drift and diffusion. The gradient energy parameter  $\gamma$  describes the first order spatial variation of the average energy through the positron swarm. These quantities are given by:

$$\varepsilon = \frac{3}{2}kT_b \left[ 1 - \sqrt{\frac{2}{3}}F(100|00) \right],$$
  
$$\gamma = \frac{3}{2}kT_b \left[ i\sqrt{\frac{2}{3}}F(100|11) \right].$$
 (14)

The components of the temperature tensor perpendicular and parallel to the electric field are, respectively:

$$T_{T} = T_{b} \left[ 1 - \sqrt{\frac{2}{3}} F(100|00;\alpha) + \sqrt{\frac{1}{3}} F(020|00;\alpha) \right],$$
(15)
$$T_{L} = T_{b} \left[ 1 - \sqrt{\frac{2}{3}} F(100|00;\alpha) - \frac{2}{\sqrt{3}} F(020|00;\alpha) + F(010|00;\alpha)^{2} \right].$$
(16)

Ratio between transverse  $T_T$  and longitudinal  $T_L$  elements of the temperature tensor is usually used as a measure of thermal anisotropy which reflects the anisotropy of the distribution function in velocity space, as discussed below. Page 5 of 10



Fig. 2. Variation of the mean energy with  $E/n_0$  in N<sub>2</sub>/CF<sub>4</sub> mixture.

#### 4 Results and discussion

#### 4.1 Preliminaries

(12)

As discussed in Sections 2 and 3, the aim of this work is to determine positron transport properties in mixtures of N<sub>2</sub> and CF<sub>4</sub>. We consider the reduced electric field  $E/n_0$ range: 0.1–350 Td (1 Td =  $10^{-21}$  V m<sup>2</sup>). The cross sections for positron scattering in N<sub>2</sub> detailed in our previous papers are used [28].

The results of a multi term solution of Boltzmann's equation are verified with those of an independent Monte Carlo simulation. Our Monte Carlo simulation code has been tested in great details under conditions when the transport of positrons is greatly affected by the Ps formation [26,27,31]. The agreement between results obtained by these two entirely independent techniques is excellent but for the reason of clarity, we show the Boltzmann equation results only.

#### 4.2 Positron transport properties in N<sub>2</sub>/CF<sub>4</sub> mixtures

# 4.2.1 Mean energy of the positrons and rate coefficient for Ps formation

In Figure 2 we show the variation of the mean energy with  $E/n_0$ . The properties of the cross sections are reflected in the profiles of the mean energy. We see that the mean energy, as expected, is greatly affected by the addition of CF<sub>4</sub> to N<sub>2</sub>. In general, the addition of CF<sub>4</sub> to N<sub>2</sub> decreases the mean energy, and the mean energy curves move to the right and one may view this as a "cooling effect". For pure CF<sub>4</sub> and mixtures of N<sub>2</sub> and CF<sub>4</sub> we see three distinct regions of transport. First, there is a region of slow rise due to large energy losses associated with vibrational excitations. Second, there is a region of sharp rise as the cross sections for vibrational excitations drop off and positrons start to rapidly gain energy from



Fig. 3. Variation of the rate coefficient for Ps formation with  $E/n_0$  in N<sub>2</sub>/CF<sub>4</sub> mixture.

the electric field. Finally, there is another region of slow rise as new inelastic channels including the Ps formation, electronic excitation and direct ionization are open. In the limit of high  $E/n_0$ , the mean energy of electrons is relatively insensitive to the gas composition. The same applies for the lowest  $E/n_0$  considered in this work where the mean energy essentially approaches to the thermal value. From Figure 2 we clearly see why CF<sub>4</sub> is such a good addition to N<sub>2</sub> when it comes to thermalizing positrons.

In Figure 3 we display the variation of the rate coefficient for Ps formation with  $E/n_0$ . The curves show the expected increase in  $k_{\rm Ps}/n_0$  with  $E/n_0$  and decrease in  $k_{\rm Ps}/n_0$  with increasing fraction of CF<sub>4</sub> in the mixture. For higher  $E/n_0 k_{\rm Ps}/n_0$  shows a weak sensitivity with respect to the composition of the mixture. Note that the values for  $k_{\rm Ps}/n_0$  vary by large orders of magnitude as the reduced electric field  $E/n_0$  is increased from below 10 Td up to 350 Td.

# 4.2.2 Drift of the positrons and negative differential conductivity

In Figure 4 we show the variation of the flux and bulk drift velocities with the applied reduced electric field  $E/n_0$ . A prominent feature of the pure  $CF_4$  and mixtures where the fraction of  $N_2$  is approximately less than 50% is the presence of a region of negative differential conductivity (NDC). This is the region in which the electron drift velocity decreases for increasing values of  $E/n_0$ . The NDC behavior for positrons and conditions leading to this phenomenon have been investigated for a range of gases, including Ar [26], N<sub>2</sub> [30], and H<sub>2</sub> [29] and H<sub>2</sub>O [31]. For all gases, except for  $N_2$ , the existence of NDC was observed only in the bulk drift velocity component with no signs of the same phenomenon in the profiles of the flux component. This was a clear indication that NDC in these gases is induced by the non-conservative nature of the Ps formation. This conclusion has been confirmed in calcu-



Fig. 4. Variation of the drift velocity with  $E/n_0$  in N<sub>2</sub>/CF<sub>4</sub> mixture.

lations where the Ps formation was treated as a conservative inelastic process; the NDC phenomenon has been removed from the profiles of the bulk drift velocity along with the differences between bulk and flux drift velocity components.

In pure  $CF_4$ , and for mixtures where the fraction of  $N_2$  is approximately less than 50%, the situation is more complex. We see from Figure 4 that both the bulk and flux drift velocities exhibit NDC. However, the origin of NDC is different for the different types of drift velocity. The NDC of the flux component is induced by the shape of cross sections as is observed for electrons in the same gas [47]. Therefore the NDC in the bulk component will also have NDC due to the shape of the cross sections but it may have an additional contribution due to the nonconservative nature of Ps formation. Following the strategy from our previous papers, we have treated Ps formation as a conservative inelastic process. As a consequence, the difference between two drift velocity components was removed but NDC effect was still present in the profiles. From the studies of electron transport in neutral gases, it is well known that NDC arises for certain combinations of elastic and inelastic cross sections in which, on increasing the electric field, there is a rapid transition in the dominant energy loss mechanism from inelastic to elastic [57,58]. The determining factor is how rapidly the ratio of the inelastic to elastic cross sections falls with the increasing mean energy/applied field. Typically, the effect is enhanced by (i) a rapidly increasing cross section for elastic collisions and/or (ii) a rapidly decreasing inelastic cross section. In the transition regime, the enhanced randomization of the directed motion decreases the drift velocity even though the mean energy increases. From Figure 4, for pure  $CF_4$  we see that NDC occurs between approximately 20 and 50 Td, where the mean energies are between 0.4and 3 eV. It is exactly in this energy range that the combination of a rapidly increasing cross section for elastic collisions and a rapidly decreasing cross section for vibrational



Fig. 5. Variation of the longitudinal diffusion coefficient with  $E/n_0$  in N<sub>2</sub>/CF<sub>4</sub> mixture.

excitation in the region of the Ramsauer-Townsend minimum in  $CF_4$  occurs which favours the development of NDC. It is clear that complexity of NDC phenomenon in  $CF_4$  follows directly from the complex energy dependence of the cross sections for positron scattering.

#### 4.2.3 Diffusion of the positrons

In Figures 5 and 6 we show the variation of the longitudinal and transverse diffusion coefficients with  $E/n_0$ . In general, it is hard to fully understand the behavior of diffusion coefficients even in pure electric field since many parallel factors affect them simultaneously. In addition to the effects of thermal anisotropy (dispersion of positrons due to thermal motion is not the same in different directions) and electric anisotropy (spatial variation of the average energy in conjunction with energy-dependent collision frequency produces differences in the average local velocities for a given direction, which act to inhibit and/or enhance diffusion in that direction), there is always the contribution of collisions and the complex energy dependence of Ps formation that even further complicate the physical picture.

Both the bulk and flux data are shown in Figures 5 and 6 with the aim of determining how sensitive diffusion is on the presence of Ps formation. First, for both  $n_0D_L$  and  $n_0D_T$  and for all mixtures we see that the bulk data are dominated by the corresponding flux data. The addition of CF<sub>4</sub> to N<sub>2</sub> increases the differences between the bulk and flux for  $n_0D_L$  while the differences between the bulk and flux for  $n_0D_T$  remain essentially unaltered. The transverse diffusion coefficient appears in general to be less sensitive to the effects of Ps formation than the longitudinal diffusion coefficient. On the other hand, for pure CF<sub>4</sub> around 100 Td, we observe deviations of more than one order of magnitude between the flux and bulk for  $n_0D_L$  which is another important manifestation of the



Fig. 6. Variation of the transverse diffusion coefficient with  $E/n_0$  in N<sub>2</sub>/CF<sub>4</sub> mixture.



Fig. 7. Variation of the ratio between transverse and longitudinal diffusion coefficients with  $E/n_0$  in N<sub>2</sub>/CF<sub>4</sub> mixture.

explicit non-conservative effects of the Ps formation processes. To study in detail the explicit effects of Ps formation on the diffusion coefficients, a comprehensive investigation of the spatially resolved data along the swarm, particularly those associated with the second-order variations of the average energy is required. This is beyond the scope of this paper and we defer the detailed discussion on the explicit influence of Ps formation on diffusion processes for positrons in mixtures of N<sub>2</sub> and CF<sub>4</sub> to a future paper.

In Figure 7 we show the variation of the ratio between the flux transverse and flux longitudinal diffusion coefficients with  $E/n_0$ . The differences between the longitudinal and transverse diffusion coefficients is evidence of the anisotropic nature of diffusion which generally follows from the interplay of energy dependent collision frequency and spatial variation of average energy along the swarm. We see that the anisotropic nature of the diffusion is enhanced as the fraction of CF<sub>4</sub> is increased. Since Page 8 of 10



Fig. 8. Variation of the ratio between transverse and longitudinal components of the temperature tensor with  $E/n_0$  in N<sub>2</sub>/CF<sub>4</sub> mixture.

the average energy increases along the swarm for all mixtures and for all applied reduced electric fields (as checked in our independent Monte Carlo simulations), it then follows that for pure  $CF_4$  and mixtures where the fraction of  $N_2$  is less than 50% the total collision frequency is a monotonically increasing function of the positron energy. For pure  $N_2$  and mixtures where the fraction of  $CF_4$  is dominated by the fraction of  $N_2$ , we see that for a certain range of  $E/n_0$  the opposite situation holds:  $n_0 D_L$  dominates  $n_0 D_T$  indicating a decrease of the total collision frequency with the positron energy in the same region. It is interesting to note that the values of the reduced electric field,  $(E/n_0)_{\text{max}}$ , for which the ratio between  $n_0 D_L$ and  $n_0 D_T$  reaches a local maximum are shifted to higher fields, and that this is not linear with increasing fraction of  $CF_4$ . It should be noted that the anisotropy of the bulk diffusion tensor is almost the same for all mixtures except for pure  $CF_4$  where there is a strong reduction of the bulk longitudinal diffusion coefficient induced by the explicit effects of Ps formation.

#### 4.2.4 The two term vs. multi term results

In Figure 8 we display the variation of the ratio between the transverse and longitudinal components of the temperature tensor with  $E/n_0$ . The anisotropy of the temperature tensor reflects the anisotropy of the distribution function in velocity space and a knowledge of its variation with  $E/n_0$  is of essential importance for studies associated with the accuracy of the two term approximation for solving the Boltzmann equation for positrons in neutral gases [29]. We observe that the anisotropy of the temperature tensor is significantly increased for increasing fractions of CF<sub>4</sub> in CF<sub>4</sub>/N<sub>2</sub> mixtures. The fields at which the  $T_T/T_L$  profiles are a minimum correspond approximately to those where the maximum error in the two term approximation occurs.



Fig. 9. Percentage difference between the two term and multi term results for the mean energy, drift velocity and diffusion coefficients for positrons in  $CF_4$ .

In Figure 9 we display the percentage difference between the two term and fully converged results for the mean energy, drift velocity and diffusion coefficients in pure CF<sub>4</sub>. Truncation at  $l_{\text{max}} = 6$  was required to achieve convergence to within 1% for all transport coefficients and properties. We observe that the mean energy and drift velocity can be in error by approximately 10% while the diffusion coefficients can have errors of the order of 100%. We see that the maximum error in the two term approximation, for all three transport coefficients occurs at about 5–10 Td. In this field region, the mean energy  $\varepsilon$  is in the range 0.1-0.2 eV, and from the plot of cross section shown in Figure 1 it is evident that at this energy the cross section for elastic collisions is almost at a minimum and the vibrational excitation processes become significant. This combined effect produces a large asymmetry in velocity space which makes the two term approximation inadequate for the analysis of positron transport data. As the cross section for elastic collisions increases rapidly with energy beyond the minimum, a decrease in the error of the two term approximation is observed in Figure 9. For higher fields, due to the influence of other inelastic channels, the accuracy of two term approximation again deteriorates, particularly for the diffusion coefficients.

## **5** Conclusion

In this paper we have presented an essentially complete set of scattering cross sections for positron in  $CF_4$ . Cross sections have been compiled from the best available data from the literature and modified using physical arguments. The main features are very large cross sections for vibrational excitations in the region of Ramsauer-Townsend minimum for elastic collisions, and a huge cross section for Ps formation exhibiting a strong energy dependence. If one wants to improve the current set of cross section for positron scattering in  $CF_4$ , then further measurements of positron swarm data are needed, including drift velocity, diffusion coefficients and rate coefficient for Ps formation. In addition, more studies of electronic excitation and dissociation are required in order to refine and normalize the existing set of cross sections. However, this set of cross sections should be sufficient for modeling of positron traps.

In addition to the collisional data, using a multi term theory for solving the Boltzmann equation, we have calculated positron transport properties, including the mean energy, drift velocity and diffusion coefficients as well as rate coefficient in pure  $CF_4$  and  $CF_4/N_2$  mixtures. Calculations have been performed under conditions critical for modeling of positron traps, and we have focussed on the way in which the positron transport properties are influenced by the explicit effects of Ps formation. The following important points have been observed: (i) the addition of  $CF_4$  to  $N_2$  decreases the mean energy and cools down the positrons; (ii) NDC phenomenon has been observed in the profiles of both the bulk and flux drift velocity; (iii) the longitudinal diffusion coefficients is more sensitive than the transverse diffusion coefficient with respect to the Ps formation; and (iv) from the profiles of the diagonal elements of the temperature tensor we have concluded that the distribution function in velocity space is highly anisotropic for certain range of the reduced electric fields which makes the two term approximation for solving the Boltzmann equation inadequate for the analysis of positron transport data.

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