

MORPHOLOGY OF PHYSICS

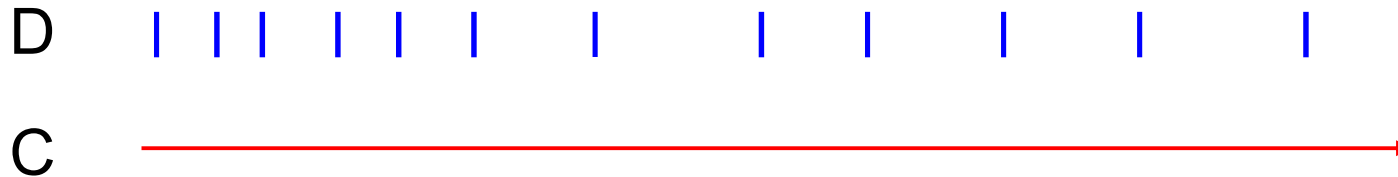
DISCRETE AND CONTINUOUS WORLDS

Preamble

This part of “**Morphology of Physics**” has been developed by Valja BOCVARSKI during years 2010-2012. It was preceded by “Extremum principles” and followed by “Quantum Geometries”.

Valja wrote the original texts in Serbian. Additional authors essentially acted rather as translators and/or commentators

What is discrete ? What is continuous ?



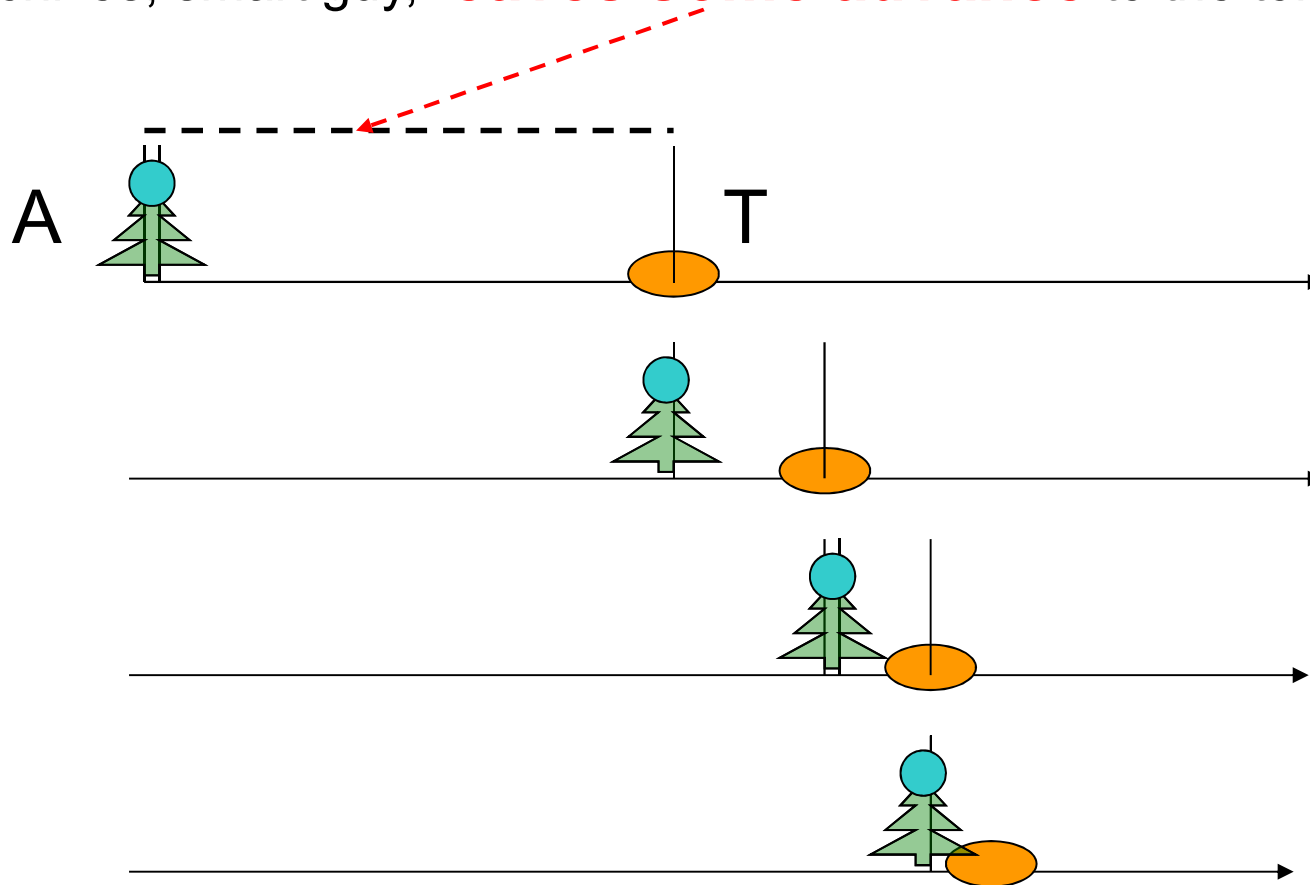
Question: is it possible, by reducing (to zero) the intervals between discrete values, to reach the continuum ? The answer is NO !

Assume the intervals are reduced by some factor λ , then by zooming the figure by a factor $1 / \lambda$, one will obtain exactly the same figure as before, independently of λ .

This is the basis of the ten Zeno paradoxes (about 5th century BC), among which Achilles and the tortoise

THE RACE BETWEEN ACHILLES AND THE TORTOISE

Achilles, smart guy, **leaves some advance** to the tortoise



Fatal error, ... it "never" ends!...

Consequence :

The movement in such a discretized continuum is impossible, *i.e.* **the movement is nothing else than an illusion...**

This conclusion is in a perfect agreement with the Parmenides conception of the world as a single continuum (5th century BC, Eleatic School). Parmenides said that « **the reality of the world is One Being, an unchanging, timeless and indestructible Whole.** »

Indeed Zeno's description of the race is perfectly logical, but **what about the interpretation ?**

Nowadays we get the impression that something is lacking, namely **the time** (which had no place in Parmenides' world), and we have to wait till the 19th century to fully understand (without contradicting Zeno) that – don't worry - Achilles will finally overpass the tortoise, by using the concept of converging series

A very simple calculation

Let $d(1), d(2), \dots, d(n), \dots$ be the successive mutual distances between A and T, and v_1, v_2 the velocities of A and T (with $r = v_2 / v_1 < 1$).

It is readily seen that $d(1) / v_1 = d(2) / v_2$ and more generally that $d(n) / v_1 = d(n+1) / v_2$. This gives:

$$d(2) = r d(1), d(3) = r d(2), \dots, d(n+1) = r d(n), \dots$$

that is **$d(n+1) = r^n d(1)$.**

Indeed, as $r < 1$, the mutual distance tends to zero when n tends to infinity.

But what about the **TIME** spent to do it ?

Till step $n+1$, A has spent a time

$$\begin{aligned} T_n &= [d(1) + d(2) \dots + d(n)] / v_1 \\ &= d(1) [1 + r^1 + r^2 + \dots + r^n] / v_1 \end{aligned}$$

For an infinite number of steps, the series is converging and T_n tends to the finite value (this is the key)

$$\mathbf{T_c = d(1) / [(1 - r) v_1]}$$

But time still runs after T_c ! Achilles, keeping his velocity v_1 , over passes the tortoise whose velocity is still v_2 .

Paradoxical Greek philosophers ! Not easy to schematize.

Example : Pythagoras made the (rational) numbers be the **discrete** basis of his world...

At the same time, he knew the straight lines - that he called “lengths without width”- which are **continuous**, and he knew also the triangle, even the rectangular one, sufficiently to imagine :

$$a^2 = b^2 + c^2$$

The general Eleatic question:

How to conciliate a continuum, which is “**One**”, with the **plurality** of things existing in the world, and their changes ?

In Ancient Greece, this question was solved by Leucippe and Democrites who imagined the ATOMS (beings) in VACUUM (non-being)

About 21 centuries later *, in Western Europe,

in particular with Descartes, this (logical) question arises again, in spite of a huge mutation in the conception of the world :

- *Monotheism and **Separation of Essence from Existence** (except in God)*
- ***The intensity of a quality** (essence) **is allowed to vary**, making a world of **time-dependent effects** – rather than a world of **invariable shapes** –*

(*) Actually, in the Arab world, this occurred about 10 centuries before, with different consequences because of a different concept of time

How did the Eleatic question arise ?

For Descartes, Existence is **the mass**.
Now appears (with the concept of time) the *dynamics*. The basic law of Descartes' dynamics is the conservation of the momentum. Besides **Matter and Space are identical**, what he called in a general sense "*l'etendue*" (the extent). In other words, Matter is a **continuum**, which immediately leads to the Eleatic contradiction.

Solutions are very similar to that given by Leucippe and Democrites centuries ago : **Atomization** or **individualization**. However this individualization will only concern the Essence of things rather than their Existence (*a distinguo* ignored by Ancient Greece).

Two famous philosophers-mathematicians and physicists (both are creators of the differential calculus) will realize it, using two very different approaches,

Newton and Leibniz,