LABORATORIJA ZA FIZIKU ATOMSKIH SUDARA

S E M I N A R

Tema: OBRADA REZULTATA MERENJA
UGAONIH RASPODELA INTENZITETA RASEJANJA
ELEKTRONA NA ATOMIMA

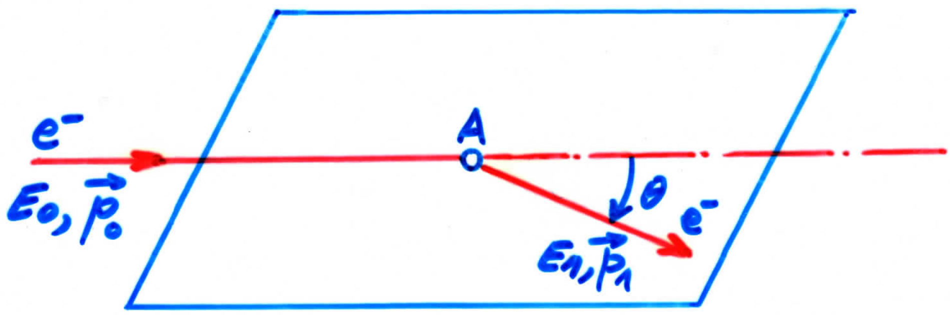
Vreme: Petak, 17. jun 1988., u 9 časova

Mesto: Sala teoretičara, soba 300

Predavač: Mr Dušan Filipović

Obrada rezultata merenja ugaonih raspodela intenziteta rasejanja elektrona na atomima

Eksperiment rasejanja



- Definicija DP-2
- Ugaona raspodela
- Relativni diferencijalni presek
- Apsolutni diferencijalni presek

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Greške merenja

- Statistička
- Slučajna
- Sistematska

STATISTIČKA GREŠKA

DOGAĐAJ X "detektovati N elektrona u vremenu Δt"

KVANTITATIVNO OBELEŽJE X JE N

X KAO SLOŽEN DOGAĐAJ OD x_i $(i = 1, 2, \dots, n)$

x_i "detektovati k_i elektrona u vremenu $\frac{\Delta t}{n}$"

$$\sum_{i=1}^{n} k_i = N$$

$k_i \sim 1 \Rightarrow x_i$ je "redak" događaj

Poisson-ova raspodela

$$P(k) = e^{-k_0} \frac{k_0^k}{k!}, \quad k_0 = np$$

p - verovatnoća pojave događaja pri jednom merenju

MATEMATIČKO OČEKIVANJE

$$M(x) = \sum_{k=0}^{N} k P(k) = \sum_{k=0}^{N} k e^{-k_0} \frac{k_0^k}{k!} = \sum_{k=1}^{N} e^{-k_0} k_0 \frac{k_0^{k-1}}{(k-1)!}$$

$$= e^{-k_0} k_0 \underbrace{\sum_{k=1}^{N} \frac{k_0^{k-1}}{(k-1)!}}_{\approx e^{k_0}} \cong k_0$$

SREDNJE KVADRATNO ODSTUPANJE

$$\sigma_x = \sqrt{k_0}$$

ZA X KAO SLOŽEN DOGAĐAJ OD NEZAVISNIH DOGAĐAJA x_i

$$M(X) = nM(x) = nk_0 = N$$

$$\sigma_X = \sqrt{\sum_{i=1}^{n} \sigma_{x_i}^2} = \sqrt{n\sigma_x^2} = \sqrt{nk_0} = \sqrt{N}$$

$$\delta_X = \frac{\sigma_X}{M(X)} = \frac{1}{\sqrt{N}}$$

ZA j NEZAVISNIH MERENJA

$$\sigma_{ST} = \sqrt{\sum_{i=1}^{j} \sigma_{X_i}^2} = \sqrt{\sum_{i=1}^{j} N_i} \qquad (1)$$

Slučajne greške

Za j ugaonih raspodela

$$X_1 , X_2 , \ldots , X_j$$

$$N_1 , N_2 , \ldots , N_j$$

$$f_1 N_1 , f_2 N_2 , \ldots , f_j N_j \quad ; \quad f_1 = 1$$

Metod najmanjih kvadrata (opšte)

$$dP = \left(\frac{1}{\sqrt{2\pi}}\right)^j e^{-\frac{1}{2}\left(t_1^2 + t_2^2 + \cdots + t_j^2\right)} dt_1 \, dt_2 \cdots dt_j$$

$$t_j = \frac{N_i - \bar{N}}{\sigma_{iG}} \qquad \bar{N} = \frac{1}{j}\sum_{i=1}^{j} N_i$$

Principni najmanjih kvadrata

$$\text{I} \quad S = \sum_{i=1}^{j} \frac{\left(f_i N_i - \bar{N}\right)^2}{\sigma_{iG}^2 + \sigma_{iP}^2} = S_{min}$$

Kako je:

$$\sigma_{iG} = \sqrt{\frac{1}{p}\sum_{l=1}^{p}\left(f_i N_{il} - N_i\right)^2} = 0 \quad , \quad p = 1$$

$$\text{II} \quad S = \sum_{i=1}^{j} \left(\frac{f_i N_i - \bar{N}}{\sigma_{iP}}\right)^2 = S_{min} \Rightarrow \bar{N}$$

$$\frac{\partial S}{\partial \bar{N}} = -2\sum_{i=1}^{j} \frac{f_i N_i - \bar{N}}{\sigma_{iP}^2} = 0 \quad , \quad \frac{\partial^2 S}{\partial \bar{N}^2} > 0$$

$$\bar{N} = \frac{1}{\sum_{i=1}^{j} \frac{1}{\sigma_{iP}^2}} \sum_{i=1}^{j} \frac{f_i N_i}{\sigma_{iP}^2} = \frac{1}{\sum_{i=1}^{j} \frac{1}{N_i}} \sum_{i=1}^{j} f_i \qquad (2)$$

• Poređenjem sa opštom formulom

$$\bar{N} = \frac{f_1 N_1 p_1 + f_2 N_2 p_2 + \cdots + f_j N_j p_j}{p_1 + p_2 + p_3 + \cdots + p_j}$$

(p_i - statistička težina), $w_i = \frac{p_i}{\sum p_i}$ - normirana st. tež.

$$\bar{N} = \sum_{i=1}^{j} w_i f_i N_i \quad , \text{ sledi}$$

$$w_i = \frac{1/\sigma_{iP}^2}{\sum_{i=1}^{j} 1/\sigma_{iP}^2} = \frac{\frac{1}{N_i}}{\sum_{i=1}^{j} \frac{1}{N_i}} \quad ; \quad \sum_{i=1}^{j} w_i = 1$$

Specijalno, za $N_1 = N_2 = \cdots = N_j = N \Rightarrow w_i = \frac{1}{j}$

Za $j < 20$ STUDENTOVA RASPODELA

$$\sigma_{SL} = \frac{1}{\sqrt{2(j-1)}} \, \frac{\sqrt{\sum_{i=1}^{j} (f_i N_i - \bar{N})^2}}{j} \qquad (3)$$

Sistematske greške

Korekcija energijske skale
u spektrima gubitaka energije

Procena: σ_{SI}

Ukupna greška

$$\sigma_u = \sqrt{\sigma_{ST}^2 + \sigma_{SL}^2 + \sigma_{SI}^2} \qquad (4)$$

Rezultat merenja

$$N = \bar{N} \pm \sigma_u \qquad (5)$$

$$N = \bar{N} \pm \sqrt{\sum_{i=1}^{j} N_i + \frac{1}{2j^2(j-1)} \sum_{i=1}^{j} \left(f_i N_i - \bar{N}\right)^2 + \sigma_{SI}^2}$$

$$\left(\bar{N} = \frac{\sum_{i=1}^{j} f_i}{\sum_{i=1}^{j} \frac{1}{N_i}} \right).$$

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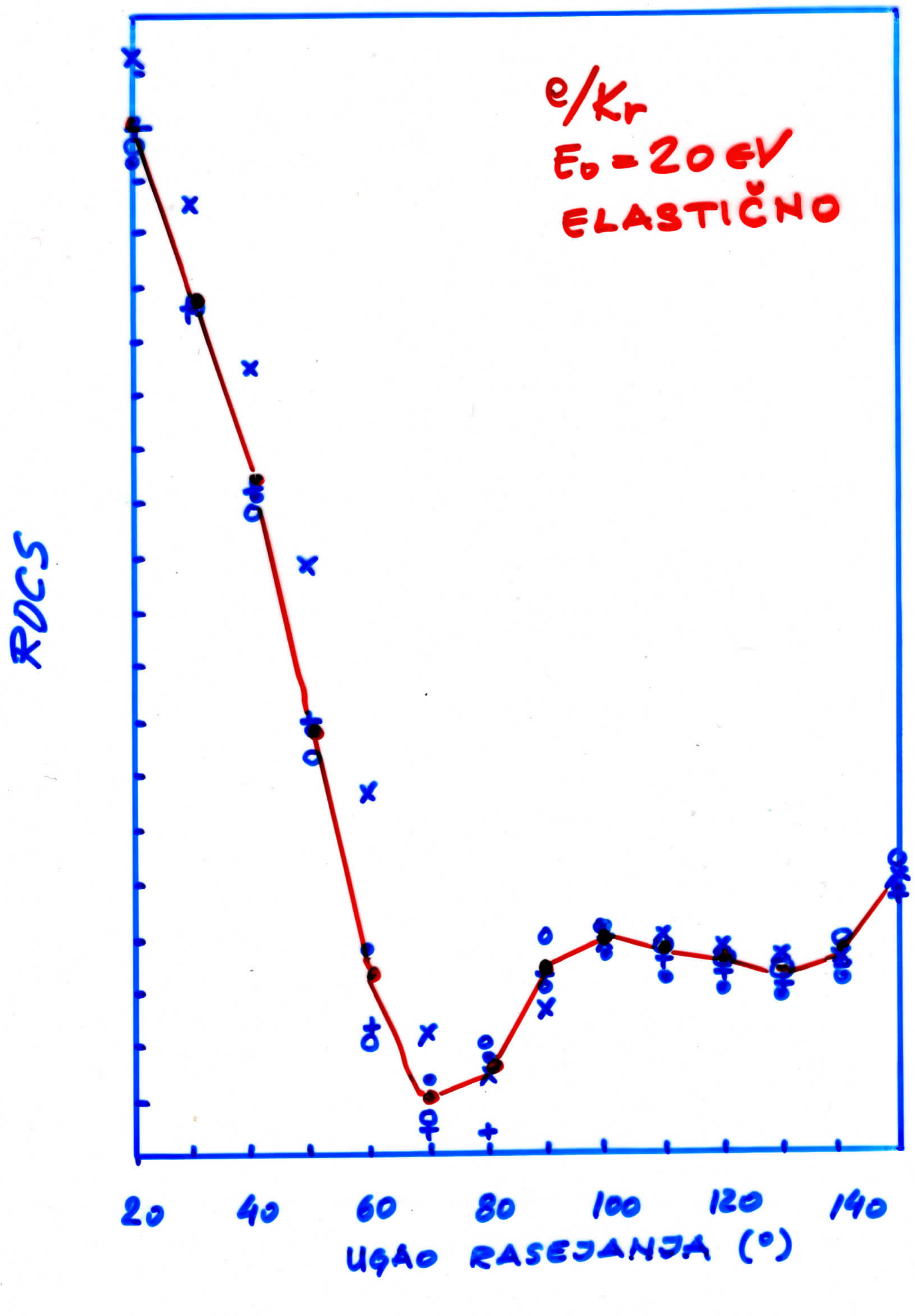
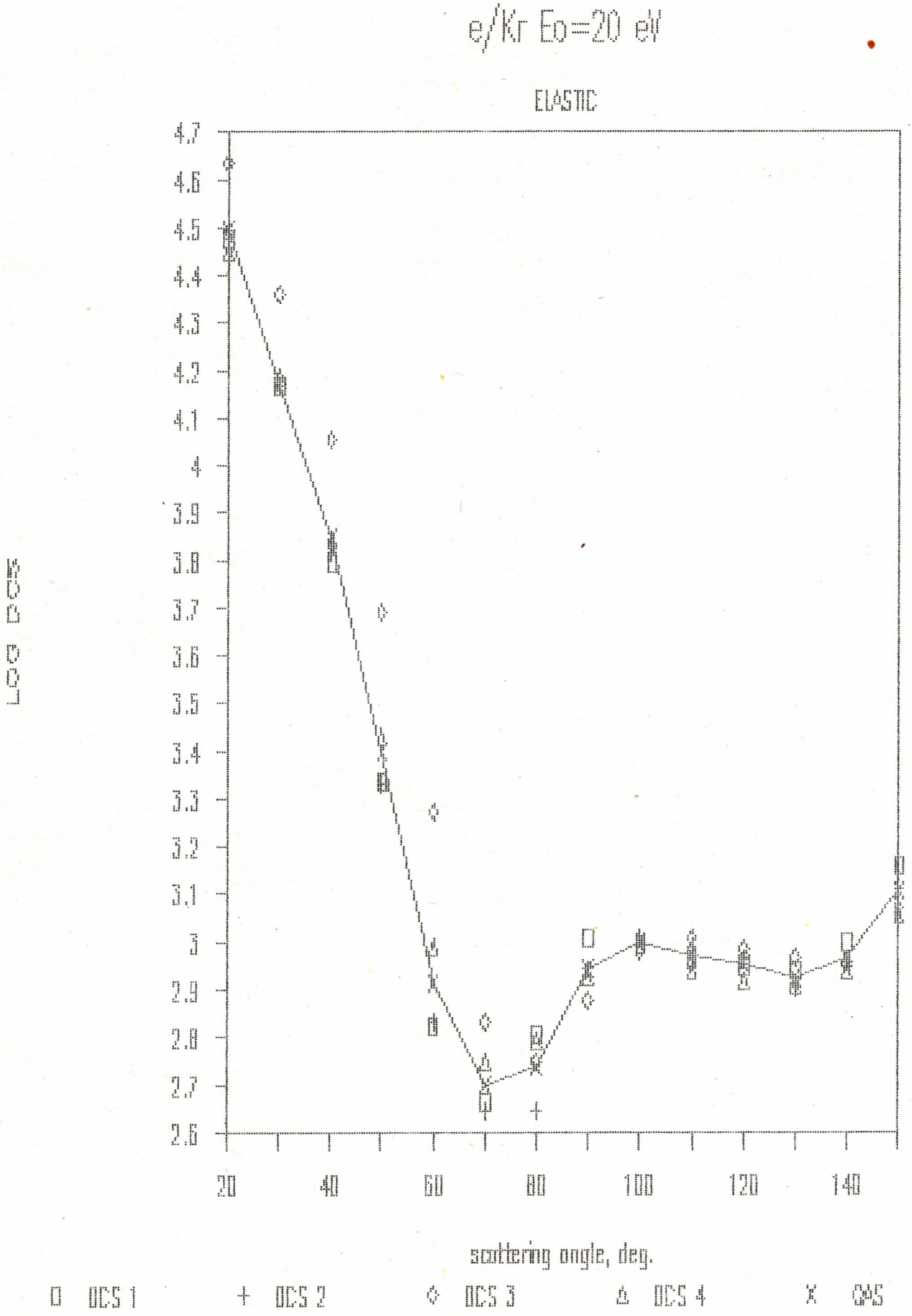
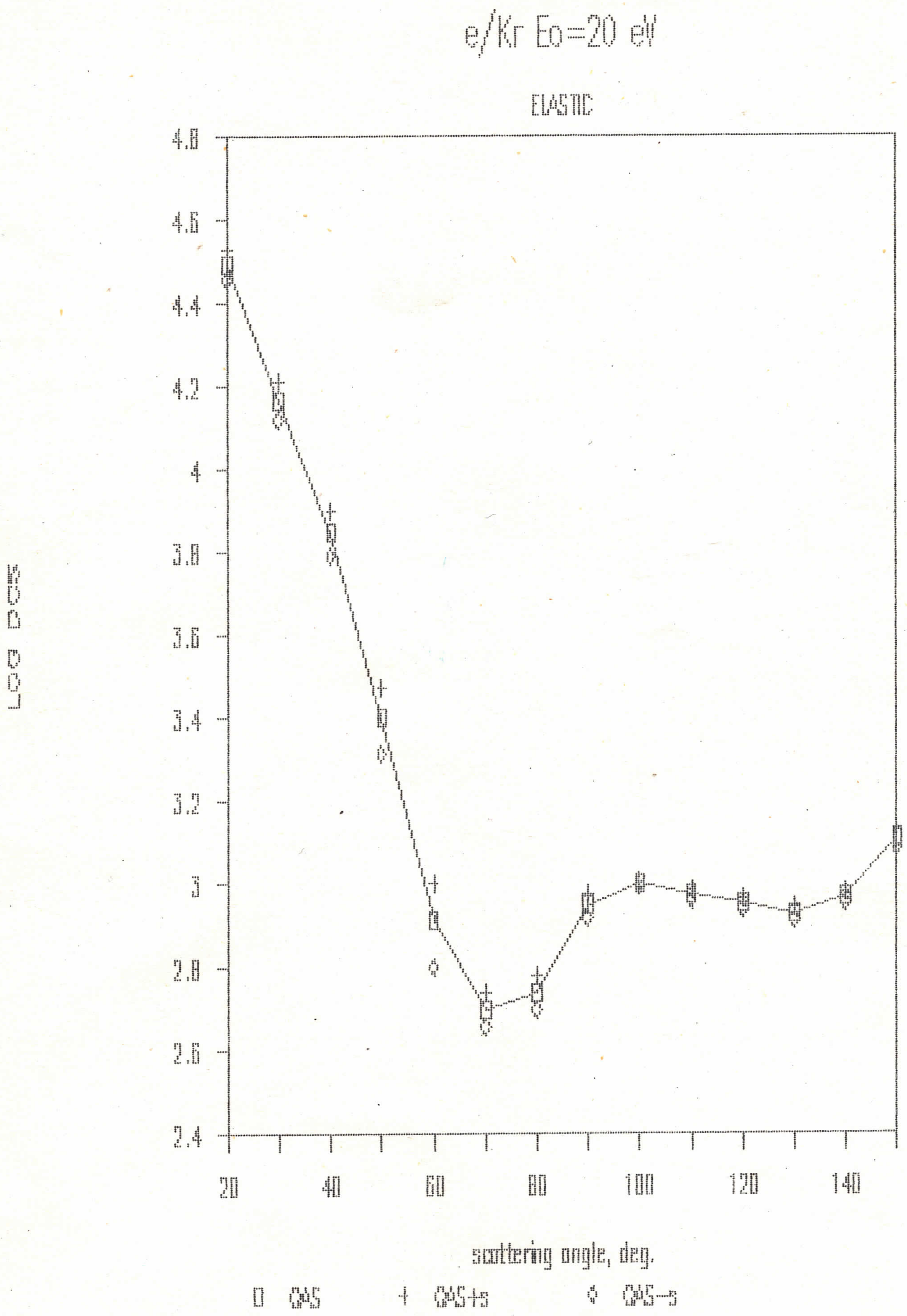
Primena mikroračunara


e/Kr
$E_0 = 20$ eV
ELASTIČNO
RDCS
20
40
60
80
100
120
140
UGAO RASEJANJA (°)


e/Kr Eo=20 eV
ELASTIC
LOG DCS
4.7
4.6
4.5
4.4
4.3
4.2
4.1
4
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
3
2.9
2.8
2.7
2.6
20
40
60
80
100
120
140
scattering angle, deg.
□ DCS 1
+ DCS 2
◇ DCS 3
△ DCS 4
X GMS


e/Kr Eo=20 eV
ELASTIC
LOG DCS
4.8
4.6
4.4
4.2
4
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
20
40
60
80
100
120
140
scattering angle, deg.
□ QAS
+ QAS+s
◊ QAS-s