Semianalytical Models of Volt–Ampere Characteristics of Diffuse Low-Current Low-Pressure Discharges

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Abstract—**We perform calculations of volt–ampere (V–A) characteristics of low-current low-pressure diffuse (Townsend) discharges in hydrogen and compare them to the experiment. The basis for the calculation is the numerical procedure for electric field calculation by solving integral equations for the surface charges. We have used analytic solutions for the homogeneous field as the initial approximation and made a cycle of calculations until the charge profile and the field were self-consistent. This procedure allows us to remove the field expansion from the physical perturbation theory of such discharges and to study the causes for negative differential resistances, oscillations, and nonlinear effects. Furthermore, we have developed a nonlinear version of the physical model of Phelps and coworkers which helps us identify the features of the ionization coefficient and secondary electron yield that give rise to nonlinear development of the V–A characteristics of Townsend regime discharges.**

Index Terms—**Breakdown, hydrogen, ionization, Townsend's theory, secondary electron yield.**

I. Introduction

REVISION of the standard Townsend's theory of gas breakdown and of the low-current low-pressure diffuse (Townsend regime) discharges has been proposed on several issues. In addition to the phenomenology of the secondary electron yield, which was revised recently [1], the most important issue that has been addressed is that of the volt–ampere (V–A) characteristics of Townsend's discharges [2]–[4]. This issue is also strongly related to the secondary electron yield and its properties [2]–[4] and the experimentally observed phenomena such as negative differential resistance, oscillations, and transition to constrictions provided abundance of information that may serve different purposes. First, one may test the modeling of secondary yields and self-consistent calculations of the fields for higher-current discharges and set appropriate benchmarks. One may also obtain the basic causes for the constriction of the discharge on the basis of simpler models of Townsend discharges rather than study the constricted discharge itself which requires more complex and, therefore, uncertain modeling. In any case, one may claim that the basic Townsend theory is not sufficient even for the low-current limit and has to be extended along the lines given in papers [1]–[4].

Phelps *et al.* have developed two models. The first one (phenomenological) was used to gain understanding of the V–A characteristic and explain the negative differential resistance, oscillations, and damping of oscillations [3], [4]. The second theory was physical in nature [3] and it was used to relate the observed properties with elementary collision and transport mechanisms. The basis of the physical theory was perturbation expansion from the low-current limit, but in this theory, the secondary electron yield was affected by the current through the change in electric field close to the cathode induced by the space charge. The expansion was implemented, in particular, for the field calculation and ion profiles and other properties were assumed to correspond to the low-current limit and to be related to the local field. The theory was extended to include nonlinear terms in a preliminary report [5].[1] In most studies of high-current discharges, secondary electron yield (and its properties such as E/N and current dependence) was used as a fitting parameter which is not satisfactory if the results are to be applied in a general way. In some studies, however a constant value taken from the binary collision data is assumed. In attempts to develop the simplest possible theory that would provide satisfactory description of the Townsend regime discharges and, thus, serve as a benchmark for theories developed for higher currents [6]–[8], it is necessary to study the role played by certain processes and, in particular, to point out how nonlinear nature of the crucial parameters affects the nonlinear behavior of the low-current discharges.

In this paper, we apply numerical procedure to solve the field exactly and we, thus, allow iterations to make corrections for spatial profiles of ions and of properties of electrons. This allows us to test the expansion model of Phelps and coworkers [3] and to make model calculations with different assumptions used for E/N (or particle energy) dependence of transport coefficients and secondary electron yields. We limit ourselves only to the dark Townsend (low-current diffuse) regime where our model is only valid. We believe that by extending the studies of the Townsend regime one may gain the knowledge of the basic causes for constriction much more easily and on the basis of a simpler model. The hydrogen is the best choice for such a study

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[1]Independently of [5], A.V. Phelps has also developed a nonlinear version of the theory of Townsend regime discharges.

since the transition to glow discharge occurs at relatively high currents of several milliamperes [9] and also since nonlinear development has been most clearly observed for such conditions.

II. Basic Theory and Numerical Procedures

A. Physical Model and Measurements of V–A Characteristics

Data for V–A characteristics used in this paper were taken from our earlier publications [4], [9]. In this paper, we present only the data for hydrogen (as it showed the most pronounced nonlinearity) and all the conditions are the same as in those references, that is: room temperature, radius of the chamber of $r = 2.25$ cm, and gap $d = 1$ cm. Thus, for these conditions, the effective current density would be $j/p^2 = 5.7\,10^{-5}$ A/(torr^2) for 1-mA current and pressure of 1.05 torr.

At the same time, the theory is based on the physical model of [3] and involves same assumptions except that field calculation is different and that multiple iterations are performed to obtain self-consistent field and density profile. The spatial distribution of ions

$$n_+(z) = \frac{j_z}{eW_{+z}}\left[1 - \exp\left(-\int_z^d \alpha(|E_z(z)|)\,dz\right)\right] \quad (1)$$

is calculated from the analytical form of ionization coefficient [2], [3]

$$\frac{\alpha}{N} = \frac{1.4\times 10^{-20}\exp(-405/E/N)}{[(10^{-4}\times E/N)^{1.5}+1]^{0.5}} \quad (2)$$

and from the knowledge of the total current j_z. The z axis is directed from cathode to anode. Here, W_{+z} is the drift velocity of the dominant ions, $E_z(z)$ is the local electric field, and E/N is the reduced effective field given in Td (1 Td $= 10^{-21}$ Vm^2). Here, ionization coefficient depends on the position through the spatially dependent electric field $E_z(z)$ which was in first iteration constant and determined by external voltage. The charged particle profile was used to calculate the electric field distribution ($E_s(z)$) due to space charge and it was combined with the external field to give the effective field that also had to satisfy the breakdown criterion

$$1 = \gamma\left(\frac{E_{\mathrm{cathode}}}{N}\right)\cdot\left(\exp\left\{\int_0^d \alpha(|E_z(z)|)\,dz\right\} - 1\right). \quad (3)$$

The secondary electron yield is dependent on the external voltage, on the space charge field, and on the current implicitly through the value of the field at the cathode. The newly calculated field was then used to determine the new profile of ions and the procedure was repeated until convergence was achieved.

In such a way, we were able to calculate the V–A characteristic of the discharge and compare it to the experimental data. In the earlier version of the physical model, it was possible to adjust secondary electron yield to follow the basic V–A characteristic. The same values gave good results in this case and the realistic current dependence of the yield gave exact representation of the V–A characteristic.

B. 1-D Solution of an Integral Equation for the Electric Field

The numerical procedure for calculating field will be given in some detail since it is somewhat different from the finite difference procedures employed in solving the Poisson equation which are commonly used.

For the purpose of this paper, we consider a one-dimensional (1-D) system with volume charges depending only on the Cartesian coordinate z. The electric field is then in the form of $\boldsymbol{E} = E\boldsymbol{i}_z$ and can be expressed in terms of the volume densities of charges as

$$E(z) = \int_0^x \frac{\rho(z')\,dz'}{2\varepsilon_0} - \int_x^d \frac{\rho(z')\,dz'}{2\varepsilon_0} \quad (4)$$

where x is the position of the field point (the point at which the electric field is evaluated), x' is the position of the source point (the point at which the charges are located), d is the distance between the electrodes, and ρ is the charge volume density.

The charge distribution may be approximated by dividing the space between the electrodes into a number of layers. Each layer is assumed to carry uniformly distributed charge, which corresponds to the so-called pulse approximation. Thus, we have

$$\rho(z) \approx \sum_{i=1}^N \rho_i f_i(z) \quad (5)$$

where $f_i(x)$ are the basis functions

$$f_i(z) = \begin{cases} 1, & \text{into } i\text{th layer} \\ 0, & \text{elsewhere} \end{cases} \quad (6)$$

where N is the total number of layers and ρ_i are constants. The approximation (5) is next substituted into (4), which yields

$$E(z) = \sum_{i=1}^{k-1}\frac{\rho_i}{2\varepsilon_0}(z_i - z_{i-1}) + \frac{\rho_k}{2\varepsilon_0}(2z - z_k - z_{k+1}) - \sum_{i=k+1}^N \frac{\rho_i}{2\varepsilon_0}(z_i - z_{i-1}) \quad (7)$$

where layer #i is confined by z_i and z_{i-1}. The field point z is assumed to be in the layer #k.

The boundary conditions require that the total charge in the system is zero. In order to provide this, there must be surface charges on the electrodes. This can be written as

$$\sum_{i=1}^N \rho_i(z_i - z_{i-1}) + \sigma_0 + \sigma_d = 0 \quad (8)$$

where σ_0 and σ_d are the surface densities of the charges on left and right electrode, respectively. The voltage U between the electrodes satisfies the following relation:

$$\int_0^d E(z')\,dz' + \frac{\sigma_0 - \sigma_d}{2\varepsilon_0}d = U \quad (9)$$

where E is the electric field from (1). The surface charges can be easily found from (5) and (6). Taking into account all charges, the final expression for the electric field is

$$E(z) = \sum_{i=1}^{k-1} \frac{\rho_i}{2\varepsilon_0}(z_i - z_{i-1}) + \frac{\rho_k}{2\varepsilon_0}(2z - z_k - z_{k+1}) - \sum_{i=k+1}^{N} \frac{\rho_i}{2\varepsilon_0}(z_i - z_{i-1}) + \frac{\sigma_0 - \sigma_d}{2\varepsilon_0}. \quad (10)$$

Equation (10) is easily converted into a numerical procedure applied to determine the field distribution. The technique described here belongs in general to the moment methods [10] and it allows us accurate calculation of the electric field for the given charge distribution and then iterative, self consistent determination of the charge profile. The procedure was tested against some already verified calculations of field profiles in high-current fields [8]. While this procedure reduces to standard finite difference techniques in a 1-D system its generalization to complex two- and three-dimensional geometries is trivial and it is general in that sense. Thus, for example, without much change we may perform similar calculations for discharges between spherical or cylindrical electrodes.

C. Nonlinear Model

It is also desirable to develop analytical or semianalytical models in order to maintain the simplicity and allow insight into which processes contribute the most to certain features of V–A characteristics. Here, we present a brief development of a nonlinear extension of the physical model of Phelps *et al.* [3], [5].

As in [3], the starting point is the Poisson equation

$$\frac{dE_{\mathrm{Sz}}}{dz} = \frac{1}{\varepsilon_0} e(n_+ - n_e) \approx \frac{1}{\varepsilon_0} e n_+ \quad (11)$$

which gives the effect of the space charge on the electric field E_S that is mainly due to slow ions and the ion density may be found from (1). The ionization coefficients and secondary electron yields may be expanded around the zero current limit of the E/N

$$\alpha(|E_z|) = \alpha(|E_{0z}|) + \alpha'(|E_z| - |E_{0z}|) + \frac{1}{2}\alpha''(|E_z| - |E_{0z}|)^2 + o[(|E_z| - |E_{0z}|)^3] \quad (12)$$

$$\gamma\left(|E_z^{\mathrm{C}}|\right) = \gamma(|E_{0z}|) + \gamma'\left(|E_z^{\mathrm{C}}| - |E_{0z}|\right) + \frac{1}{2}\gamma''\left(|E_z^{\mathrm{C}}| - |E_{0z}|\right)^2 + o\left[\left(|E_z^{\mathrm{C}}| - |E_{0z}|\right)^3\right] \quad (13)$$

where superscript C denotes the value of the field at the cathode.

Here, $\alpha_0 = \alpha(E_{0z})$ denotes the ionization coefficient at the E/N corresponding to the breakdown voltage in the limit of zero current. The properties of secondary electron yield and ionization coefficient may be used then to solve the Poisson equation coupled with the breakdown condition (3). First, however we have to define the following terms:

$$\delta V_{\mathrm{S}}^{(1)} := -\int_0^d \left(E_{\mathrm{Sz}}(z) - E_{\mathrm{S}}^{\mathrm{C}}\right) dz = -\frac{1}{\varepsilon_0}\frac{j_z}{W_{+z}} f_1(\alpha_0, d) \quad (14)$$

$$\delta V_{\mathrm{S}}^{(2)} := -\int_0^d \left(E_{\mathrm{Sz}}(z) - E_{\mathrm{S}}^{\mathrm{C}}\right)^2 dz = -\frac{1}{\varepsilon_0^2}\left(\frac{j_z}{W_{+z}}\right)^2 f_2(\alpha_0, d) \quad (15)$$

where

$$f_1(\alpha_0, d) = \frac{1}{\alpha_0}\int_0^d [\alpha_0 z - \exp(\alpha_0(z-d)) + \exp(-\alpha_0 d)]\, dz \quad (16)$$

$$f_2(\alpha_0, d) = \left(\frac{1}{\alpha_0}\right)^2 \int_0^d [\alpha_0 z - \exp(\alpha_0(z-d)) + \exp(-\alpha_0 d)]^2\, dz. \quad (17)$$

We have solved the distribution of the field due to the space charge as given by (11) while implementing the ion-density profile, as given by (1). In all equations, series expansions were performed for ionization and secondary yield coefficients as given by (12) and (13). Substituting (12) and (13) into the discharge maintenance condition (3) and keeping only the terms to the second order in E, leads after some algebra to the equation of the second order in δV

$$\delta V := -\int_0^d E_{\mathrm{Sz}}(z)\, dz = \delta V_{\mathrm{S}} - E_{\mathrm{S}}^{\mathrm{C}} d = -\frac{1}{2k_1}\left\{ k_2 + k_3 \delta V_S^{(1)} \mp \left[k_2^2 + (2k_2k_3 - 4k_1k_4)\delta V_S^{(1)} + \left(k_3^2 - 4k_1k_5\right)\left(\delta V_S^{(1)}\right)^2 - 4k_1k_6\delta V_S^{(2)} \right]^{1/2} \right\} \quad (18)$$

where the coefficients k are given by

$$\begin{aligned} k_1 &= d\gamma_0^2\alpha''\exp(\alpha_0 d) + d^2\gamma_0^2(\alpha')^2\exp(\alpha_0 d) + 2d\gamma_0\gamma'\alpha'\exp(\alpha_0 d) + \gamma'' \\ k_2 &= 2d^2\gamma_0^2\alpha'\exp(\alpha_0 d) + 2d\gamma' \\ k_3 &= -2d\gamma_0\alpha'\gamma'\exp(\alpha_0 d) - 2\gamma'' \\ k_4 &= -2d\gamma' \\ k_5 &= \gamma'' - d\gamma_0^2\alpha''\exp(\alpha_0 d) \\ k_6 &= -d^2\gamma_0^2\alpha''\exp(\alpha_0 d). \end{aligned} \quad (19)$$

If we perform numerical calculations and compare the terms, (18) may be simplified to give

$$\delta V \approx \delta V^{(1)} + \left(\frac{j_z}{\varepsilon_0 W_{+z}}\right)^2 \left[\frac{k_6}{k_2} f_2(\alpha_0, d) + \left(\frac{k_3^2}{4k_1k_2} - \frac{k_5}{k_2}\right)(f_2(\alpha_0, d))^2\right], \quad (20)$$

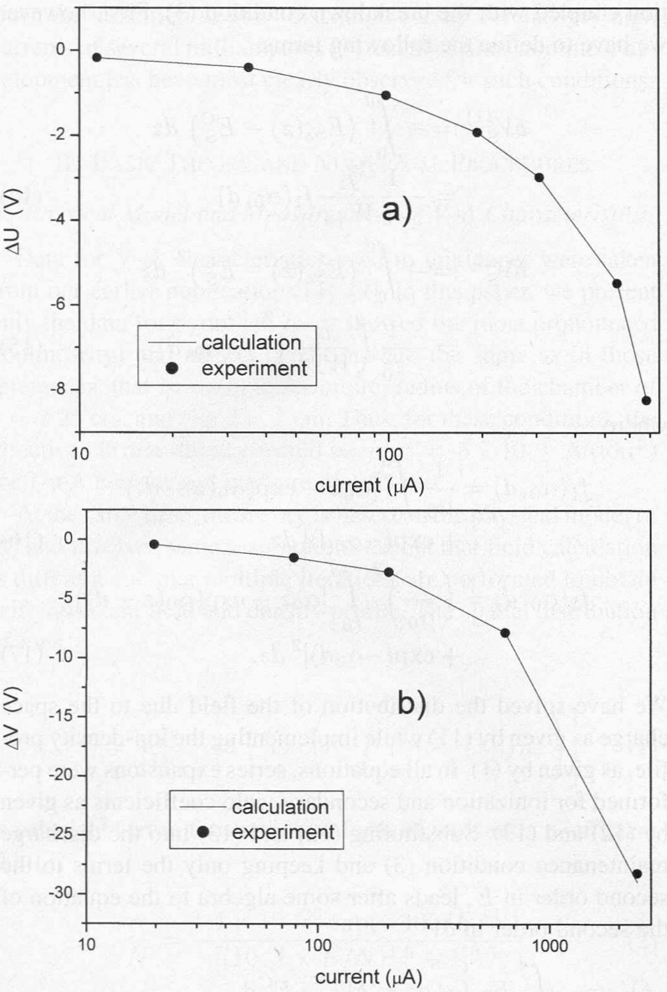

Fig. 1. Current dependence of the difference between the breakdown voltage and the voltage at the given current (DV) at (a) 1.05 and (b) 0.825 torr cm in hydrogen.

and for the conditions of our experiment, it may be simplified even further

$$\delta V \approx \delta V^{(1)} + \left(\frac{j_z}{\varepsilon_0 W_{+z}}\right)^2 \left[\frac{k_6}{k_2} f_2(\alpha_0, d)\right]. \quad (21)$$

Thus, one can associate the nonlinear behavior with the nonlinear components of both ionization coefficient and secondary electron yield.

III. Computed V–A Characteristics and Discussion

One example of calculations of the V–A characteristics is given in Fig. 1(a), where we show an excellent agreement of the current dependence of the change in the voltage (the difference between the zero current limit and the actual breakdown voltage) for $pd = 1.05$ torr cm. In that example, the secondary electron yield required to fit the V–A characteristic by the physical model of Phelps *et al.* was used without further modification. If, however, we repeat the same procedure for $pd = 0.825$ torr cm [see Fig. 1(b)] where the Townsend regime may be extended to several milliamperes, the discrepancy becomes visible but, in that case, the change of voltage becomes a significant part of the breakdown voltage, which is of the order of 400 V in both cases.

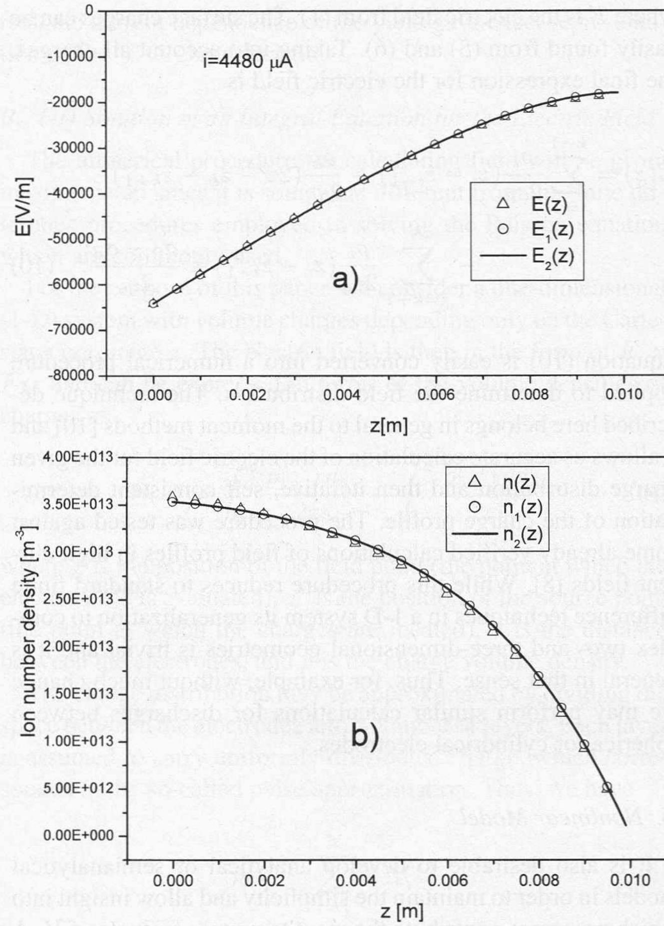

Fig. 2. Spatial distribution of (a) electric field and (b) ion number density for $pd = 0.825$ torr cm and for 4.48 mA in hydrogen.

From the calculations, we may make a conclusion that analytic solution of Poisson equation implemented in [3] and the related assumption of the very weakly perturbed ion-density distribution is quite good for the present range of circumstances. In Fig. 2, we show the calculated spatial profile of electric field. The field from the theory in [3] $E(z)$ is used to calculate the resulting ion density distribution $n_1(z)$. The new distribution is used to recalculate the field profile $E_1(z)$ which gives result to the new ion density profile $n_2(z)$. It turns out that in most cases, the differences are very small and impossible to see in the graphs and even for the highest current, as shown in Fig. 2, differences are small and a single iteration is sufficient for the procedure to converge. However, under those circumstances, it is quite probable that the assumption that electrons do not contribute to the space-charge effect is increasingly inaccurate and perhaps leads to disagreement with experiment as seen in Fig. 1(b).

Now we can proceed to check how different properties of two key parameters affect the V–A characteristics. First, we make calculations with α given by (2) and we adjust properties of the secondary electron yield. The results are shown in Fig. 3. It is observed that linear approximation of the secondary electron yield on the current density is a very good approximation explaining the low-current limit of negative differential conductivity. Close to the transition to constriction, as expected, higher order terms become important (see Fig. 4) probably due to nonlinear axial and radial space charge effects. Assuming a constant

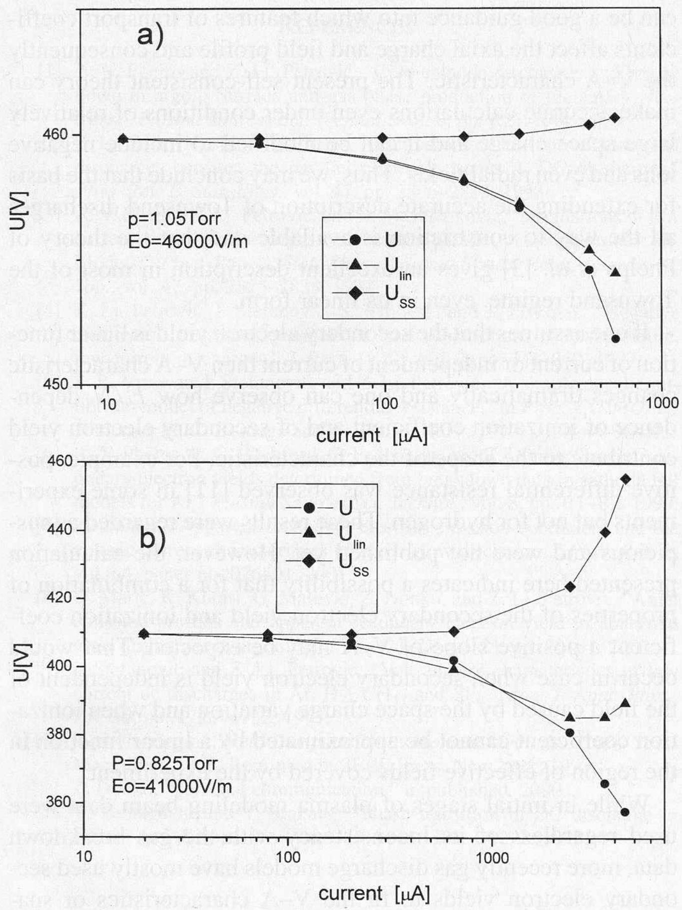

Fig. 3. V–A characteristics of hydrogen discharges at (a) 1.05 and (b) 0.825 torr. With U we denote calculated values with γ adjusted to fit experimental characteristic, with U_{lin} we choose γ_{lin} with linear dependence on current and with U_{ss} we denote calculation with a constant γ_{SS} required to give a correct breakdown voltage.

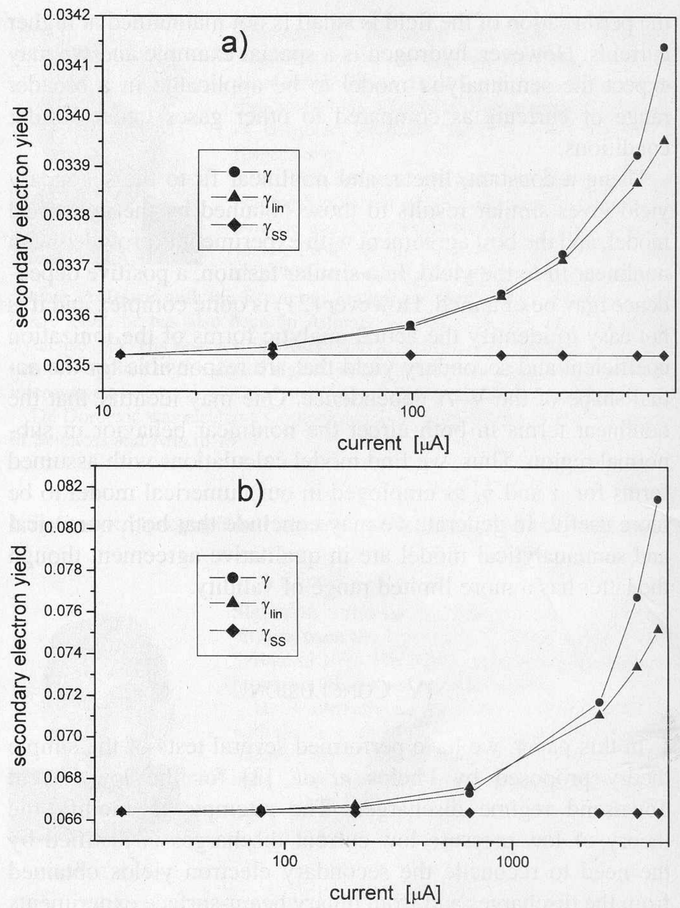

Fig. 4. Current dependence of the secondary electron yield at (a) 1.05 and (b) 0.825 torr.

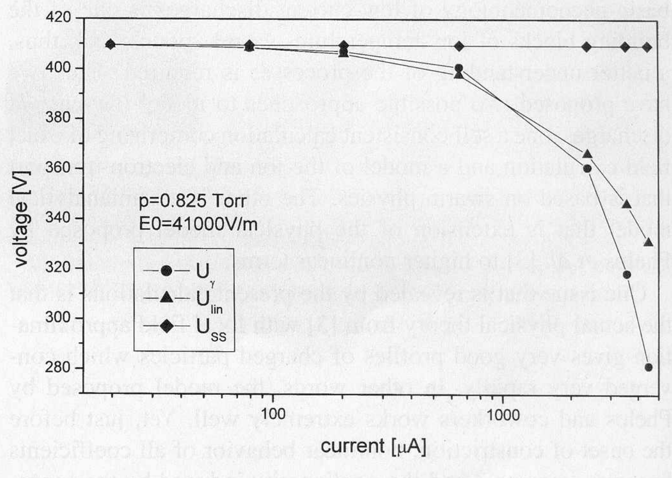

Fig. 5. V–A characteristics of the discharge in hydrogen at 0.825 torr with linear approximation for alpha around the breakdown E/N. U_γ denotes the calculated voltage with the same secondary yield that was used to fit the experimental data with realistic alpha, as shown in Fig. 4(b), and we also perform calculations for linear (lin) and constant (SS) secondary electron yield.

secondary electron yield actually produces an increasing V–A dependence. This is due to the nonlinear dependence of ionization coefficient on E/N. Thus, some claims that positive slopes of the V–A characteristics have been observed [11] in helium may be caused by relative independence of the secondary yield on current under the conditions of that experiment.

If we take the ionization rate to increase linearly around the mean value at the low-current limit of the field, a constant V–A dependence is observed for a constant secondary yield. For linear and nonlinear secondary yields (see Fig. 5), we also see a significant change in the V–A characteristics of the discharge which are all results of the change in the nature of ionization coefficient. It appears that the dependence of the ionization coefficient on E/N near the saddle point [see (20)] may affect further development of the V–A dependence and the transition to constriction as has been suggested by some authors [12].

If we apply a semianalytic model as expected, the linear theory gives excellent agreement with the experiment as our model secondary electron yield was obtained by fitting the experimental data and using this theory. As results are extremely sensitive on the secondary electron yield, the nonlinear theory does not give a very good fit, similar by somewhat worse than the departure shown in the results of the numerical model. However, the fit improves as nonlinear terms for α and γ are included in the calculation. Comparing the results of the numerical and semianalytic model, however, we can see that first one fits the data almost to the transition to glow regime that is up to 2 mA for 0.825 torr cm, while the semianalytic model is able to make a similar fit up to 300 μA. That means that the primarily assumption of the semianalytic model, that the perturbation of the field is small is not maintained at higher currents. However, hydrogen is a special example and we may expect the semianalytic model to be applicable in a broader range of currents as compared to other gases under similar conditions.

Using a constant, linear, and nonlinear fit to the secondary yield gives similar results to those obtained by the numerical model, and the best agreement with experiment is provided with nonlinear fit to the yield. In a similar fashion, a positive dependence may be obtained. However (21) is quite complex and it is not easy to identify the actual analytic forms of the ionization coefficient and secondary yield that are responsible for the actual shape of the V–A dependence. One may identify that the nonlinear terms in both affect the nonlinear behavior in subnormal region. Thus, we find model calculations with assumed forms for α and γ, as employed in our numerical model to be more useful. In general, we may conclude that both numerical and semianalytical model are in qualitative agreement though the latter has a more limited range of validity.

IV. Conclusion

In this paper, we have performed several tests of the simple theory proposed by Phelps *et al.* [3] for the low-current Townsend regime discharges. The attempts to modify the theory of low-pressure low-current discharges is justified by the need to reconcile the secondary electron yields obtained from the discharges and from binary beam-surface experiments [1] in order to enable application of the available data in modeling of higher-current discharges where knowledge of the secondary yield cannot be obtained directly. In addition, the basic phenomenology of low-current discharges is one of the building blocks of low-temperature plasma models and, thus, a better understanding of the processes is required. Thus, we have proposed two possible approaches to model low-current discharges, one a self consistent calculation comprising of exact field calculation and a model of the ion and electron transport that is based on swarm physics. The other is a semianalytical model that is extension of the physical model proposed by Phelps *et al.* [3] to higher nonlinear terms.

One issue that is revealed by the present calculations is that the actual physical theory from [3] with local field approximation gives very good profiles of charged particles which converged very rapidly. In other words, the model proposed by Phelps and coworkers works extremely well. Yet, just before the onset of constriction, nonlinear behavior of all coefficients becomes important and the nonlinearity induced by the space-charge profile becomes quite large leading to self organized transition to the constricted regime It is certain that in the extreme region close to constriction radial fields also become significant and they affect the discharge. Fitting this regime would yield a nonlinear development of the secondary electron yield if the present theory which in essence is single dimensional, is applied. That means that one should be aware that due to large uncertainties in the secondary electron yields, influence of completely unrelated effects may be assigned to it and in this case it may be the radial field development or even the nonlinearities in the axial field and ion transport. The present nonlinear theory can be a good guidance into which features of transport coefficients affect the axial charge and field profile and consequently the V–A characteristic. The present self-consistent theory can make accurate calculations even under conditions of relatively large space charge and it can be modified to include negative ions and even radial fields. Thus, we may conclude that the basis for extending the accurate description of Townsend discharges all the way to constriction is available and that the theory of Phelps *et al.* [3] gives an excellent description in most of the Townsend regime, even in its linear form.

If one assumes that the secondary electron yield is linear function of current or independent of current then V–A characteristic changes dramatically and one can observe how E/N dependence of ionization coefficient and of secondary electron yield contribute to the shape of the characteristic. For example, positive differential resistance was observed [11] in some experiments but not for hydrogen. These results were regarded as suspicious and were not published yet. However, the calculation presented here indicates a possibility that for a combination of properties of the secondary electron yield and ionization coefficient a positive slope of V–A may be expected. That would occur in case when secondary electron yield is independent of the field caused by the space charge variation and when ionization coefficient cannot be approximated by a linear function in the region of effective fields covered by the experiment.

While in initial stages of plasma modeling beam data were used regardless of its inconsistency with the gas breakdown data, more recently gas discharge models have mostly used secondary electron yields to fit the V–A characteristics or spatial emission profiles [6]–[8]. However, Phelps and Petrović [1] have shown that the dominant feedback mechanism at high pressures is photo effect at the cathode due to resonant photons and at low pressures is fast neutral ionization in gas phase. In complex, higher power plasmas it may not be possible to maintain a linear correspondence between these effects and ion flux and, thus, to assign these effects to the yield associated with the flux of ions. Thus, it is desirable to understand how secondary yields depend on different properties of the cathode and of the discharge and to be able to model the maintenance of the discharge on the basis of elementary processes in the same way that it was done for the breakdown in argon [1].

Application of the assumption (due to Townsend) that secondary electrons from the cathode originate from ion impact is actually acceptable for Townsend regime provided that one does not require quantitative comparisons with binary collision data. That is so because the fluxes of ions, photons, and metastables are proportional to the electron flux and stationary. Then, one may use fitting of the breakdown and V–A data to obtain stationary and local differential properties of the effective secondary yield. Due to this property, Townsend discharges close to constriction may be regarded as a benchmark against which all plasma models should be tested.

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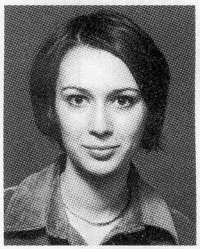

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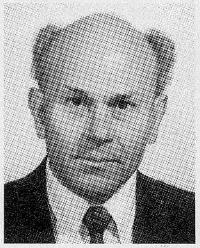
Her research interests include numerical electromagnetics applied to electrostatics, antennas, microwave circuits, and plasma etching.



Antonije R. Dordević was born in Belgrade, Yugoslavia (now Serbia), on April 28, 1952. He received the B.Sc., M.Sc., and D.Sc. degrees from the School of Electrical Engineering, University of Belgrade, Belgrade, in 1975, 1977, and 1977, respectively.

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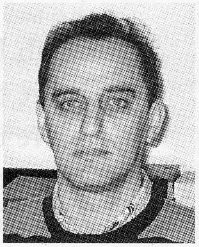
I. Stefanović, photograph and biography not available at the time of publication.



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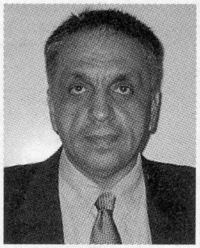

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